



QUANTIFICATION OF CHAOTIC DYNAMICS IN RAINFALL VARIABILITY OF SOME SELECTED STATIONS IN NIGERIA

Adelaja, A. D. ¹; Fowodu, T. O. ¹; Bamidele, K. J.¹; Adebisin, B. O.²; Oloyede, E. S.¹ & Falayi E .O. ¹

¹ Department of Physics, Tai Solarin University of Education, Ijagun, Ogun State, Nigeria.

²Department of Physical Sciences, Hillside University of Science and Technology, Oke-Mesi 360100, Nigeria.

*Corresponding author Email address: ayodeleenochadelaja@gmail.com

Abstract

Effectively managing domain knowledge acquisition challenges has been a major and persistent Chaotic quantification of monthly rainfall data for 20 stations in Nigeria between 1901 and 2020 was investigated. The complexity in rainfall variability is analysed using chaotic nonlinear techniques of both average mutual information (AMI) and false nearest neighbors (FNN) which were used in calculating time delay (τ) embedded dimension (m) respectively. The Lyapunov exponents (LE) as a chaotic quantifier is used to express the level of complexity in the nonlinear dynamics based on embedding parameters. The LEs which are all positive and ranges from 0.25 to 0.65 across all the stations signifying the chaotic nature of rainfall data. Gombe and Kaduna had the highest values of LE, while the lowest LE was observed at Ekiti, Ondo, Osun, Enugu and Imo. This study showed that the inter-tropical convergence zone (ITCZ) have influence on the stability and irregularity conditions of rainfall owing to alteration in the degrees of freedom created over an outside stochastic driver.

Keywords: Nonlinear time series, Rainfall, Lyapunov exponent, Average Mutual Information and False Nearest Neighbors.

1.0 Introduction

Rainfall plays an important role in regional water circulation and water balance being one of the primary requirements for calculating and analyzing various phenomena in hydrology, metrology, and agriculture. Global economy growth, particularly in agricultural planning and management requires a precise and accurate

analysis of rainfall distribution of the location over the time. Insufficient rainfall results in drought, while excessive rainfall generates floods. Both situations can have disastrous consequences.

Rainfall is influenced by climatic events and its relationships between the atmosphere, ocean, and ground surface across many time scales (Wallace and Hobbs, 2006; Falayi *et*

al., 2022). Rainfall behavior has been studied using a variety of linear and nonlinear methods, including precipitation series analysis (Brunetti *et al.*, 2004), spectrum analysis of rainfall data (Poornima and Vinayakam, 2018), and temporal analysis of rainfall (De Luis *et al.*, 2000; Estrela *et al.*, 2000; Agnese *et al.*, 2002). Sen's Slope Estimator, the Mann-Kendal Test, and simple regression were used to analyze trends in the Wami River Basin's annual and seasonal rainfall time series from 1983 to 2017 and their impact on rural communities' water supply services (Sekela and Manfred, 2019).

Numerous factors have influenced rainfall and other water science processes, resulting in significant spatiotemporal variability and seemingly random but non-random properties. Because of these intricacies, it is challenging for conventional mathematical models to effectively mimic and predict these water science processes. However, the advent of chaos theory offers fresh perspectives on this extremely complex system (Wei *et al.*, 2014; Yan *et al.*, 2015). Instead of depending just on basic statistics, understanding the actual rainfall distribution over a given area has improved a number of applications of rainfall data.

Investigation of the uncertainties and complexities of some atmospheric parameters dynamics has been successfully carried out using Chaos theory (Sivakumar, 2000; Adelaja *et al.*, 2021), the chaotic techniques used in identifying the hidden dynamics of the deterministic dynamical mechanisms are phase-space reconstruction and embedding of a single-dimensional time series in a multi-dimensional phase-space. In investigating the chaotic structures of some

natural occurrence, many researchers have employed the concept of chaos theory to examine, predict and reveal the hidden dynamics of those natural systems: hydrology (Sivakumar, 2000), solar wind parameters and geomagnetic indices (Falayi *et al.*, 2020), Machine learning models for prediction of rainfall (Ojo and Ogunjo, 2022), sunspot prediction (Park *et al.*, 1996; Sarp *et al.*, 2018), solar irradiance analysis (Adelaja *et al.*, 2021). Falayi *et al.* (2022) analysed the chaotic behaviour of monthly rainfall data of some West African countries using wavelet transformation analysis and time series techniques. Adewole *et al.* (2020) investigated the chaotic time series analysis of meteorological parameters in some selected stations in Nigeria and the study concluded that the meteorological parameters used were good tools for modeling weather-predicting systems with different dynamic variables for the selected locations.

In this research, we investigated the variation of rainfall across Nigeria (Akwa-Ibom, Baysa, Cross River, Ondo, Ekiti, Osun, Anambra, Enugu, Imo, Lagos, Abuja, Benin, Kogi, Kwara, Jijawa, Kaduna, Kano, Borno, Gombe and Yobe States) from 1901 to 2020 using nonlinear time series. This paper is structured as follows. Section 2 describes the observational data and methodology for the rainfall distribution. In section 3, we apply nonlinear time series techniques to investigate the dynamics of rainfall variation across Nigeria. Discussion of the results is presented in section 4 and section 5 provides the Conclusion.

2.0 Materials and methods

In this study, monthly mean rainfall data for 120 years (1901–2020) from some stations

across major cities in Nigeria were considered. The time series data which was compiled to be monthly variations for each station are collected from the World Bank Data Group ([http://sdwebx.worldbank.org/climateportal/index.cmf?page=downloadscaled_data_download & menu=historical](http://sdwebx.worldbank.org/climateportal/index.cmf?page=downloadscaled_data_download&menu=historical)). Chaos concept was used to examine the dynamics of the monthly rainfall variation from those

stations. The hidden dynamics in the system due to its nonlinearity can be described by chaos theory. Chaotic theory due to its deterministic nature has the potentials to reveal any inconsistency in the time series and also points out if long-term prediction for the system is realistic. Table 1 depicts the geographical coordinates of the various observatory.

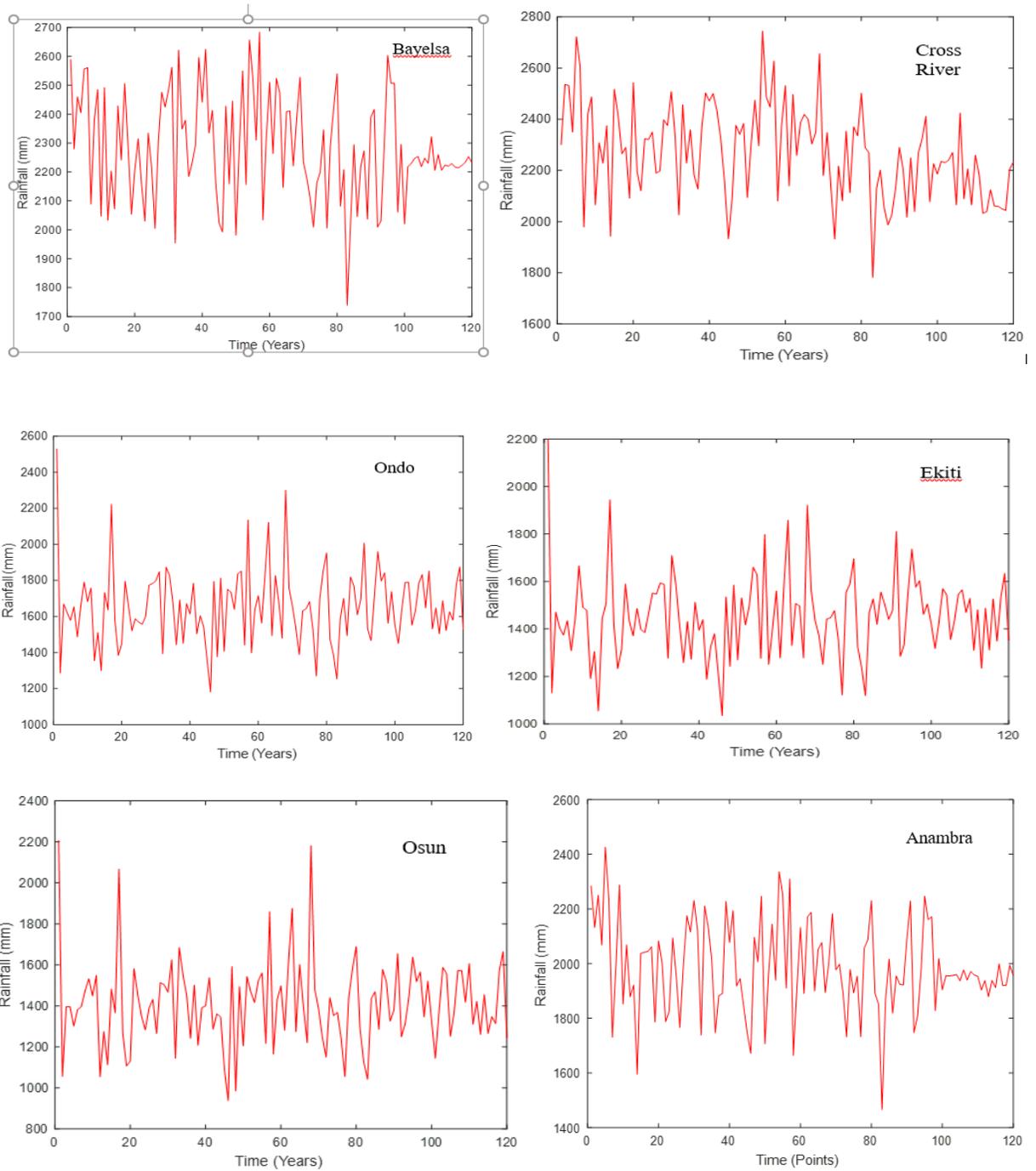
Table 1: Geographical Coordinates of the Observatory.

STATION	CODE	LATITUDE	LONGITUDE	GEO-POLITICAL ZONE
Abuja	ABJ	9.07	7.39	North-Central
Akwa-Ibom	AKI	4.94	7.84	South-Southern
Anambra	ANA	6.22	6.94	South-Eastern
Bayelsa	BAY	4.76	6.02	South-Southern
Benue	BNU	7.35	8.78	North-Central
Borno	BOR	12.19	13.31	North-Eastern
Cross-River	CRR	5.87	8.52	South-Southern
Ekiti	EKI	7.74	5.27	South-Western
Enugu	ENU	6.55	7.41	South-Eastern
Gombe	GOM	10.38	11.21	North-Eastern
Imo	IMO	5.59	7.07	South-Eastern
Jigawa	JIG	12.32	9.51	North-Western
Kaduna	KAD	10/38	7.85	North-Western
Kano	KAN	11.84	8.54	North-Western
Kogi	KOG	7.79	6.69	North-Central
Kwara	KWA	8.84	4.67	North-Central
Lagos	LAG	6.53	3.58	South-Western
Ondo	OND	7.02	5.06	South-Western
Osun	OSU	7.55	4.50	South-Western
Yobe	YOB	12.12	11.51	North-Eastern

3.0 Monthly Rainfall Variations

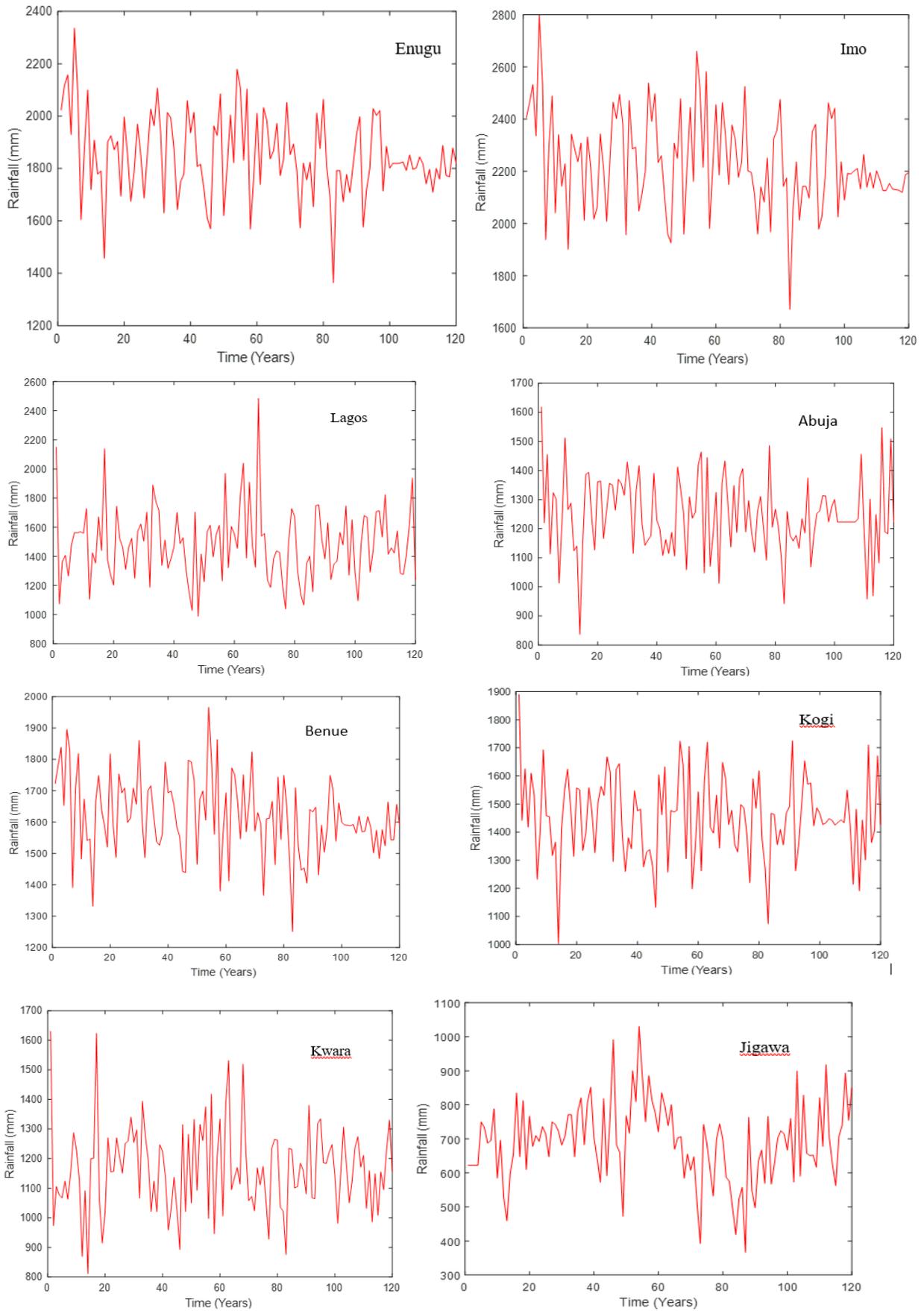
Rainfall is an atmospheric phenomenon restraining solar radiation at the ground surface. Figure 1(a – t) shows variation plots of monthly distributions of rainfall across Nigeria between 1901 and 2020. Figure 1(a - t) which shows the variation plots for Abuja, Bayelsa, Cross River, Akwa Ibom, Ondo, Ekiti, Osun, Anambra, Enugu, Imo, Lagos, Benue, Kogi, Kwara, Jigawa, Kaduna,

Kano, Borno, Gombe and Yobe state respectively displaces highly irregular dynamics (pattern) all through the stations.



QUANTIFICATION OF CHAOTIC DYNAMICS IN RAINFALL VARIABILITY OF SOME SELECTED STATIONS IN NIGERIA

Adelaja, et al.



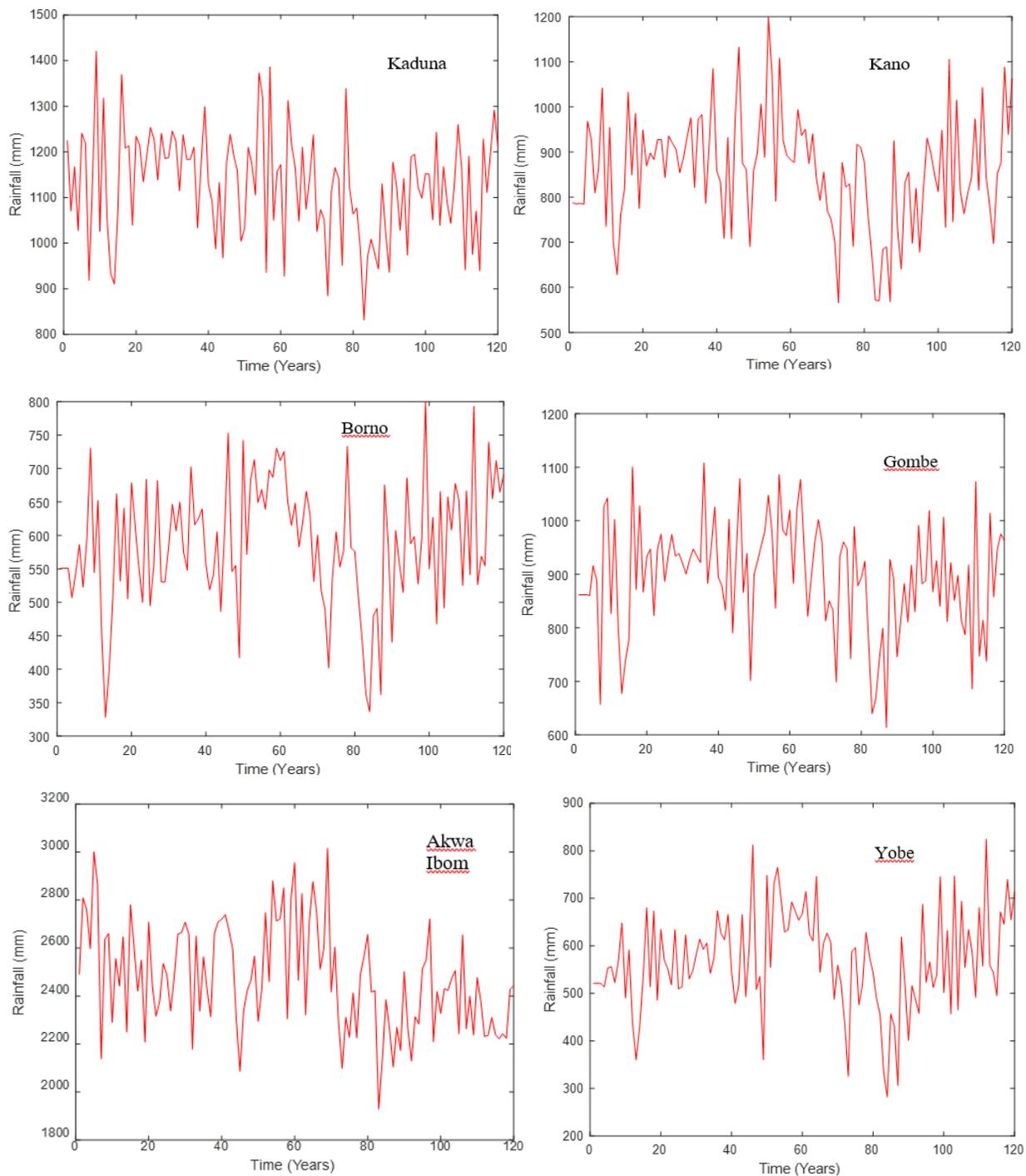


Figure 1: Rainfall variability for (a) Bayelsa, (b) Cross river, (c) Ondo (d) Ekiti (e) Osun (f) Anambra (g) Enugu (h) Imo (i) Lagos (j) Abuja (k) Benue (l) Kogi (m) Kwara (n) Jigawa (o) Kaduna (p) Kano (q) Borno (r) Gombe (s) Akwa Ibom (t) Yobe respectively.

3.1 Dynamics of Rainfall Variability using Nonlinear Techniques

Non-linear time series techniques such as Lyapunov exponent (LE), false nearest neighbour (FNN) and average mutual information (AMI) are tools used in

revealing the hidden structures of a univariate data and consequently show the dimensional chaotic behaviour of such system. Lyapunov exponent (chaotic quantifier) identifies the nonlinear chaotic behaviour in a dynamic system. Lyapunov exponents are long time average exponential rates of divergence or convergence of nearby states in the phase space. If a system

has at least one positive Lyapunov exponent, then it is assumed to be chaotic.

3.2 Phase space Reconstruction

The time series is assumed to be generated by a nonlinear dynamic system with d degrees of freedom. It is therefore necessary to construct an appropriate series of state vectors $X^d(t)$ with delay coordinates in the d -dimensional phase space:

$$X^d(t) = [X(t), X(t+\tau), \dots, X(t+(d-1)\tau)] \quad (1)$$

where t is an appropriate time delay. The trajectory in the phase space is defined as a sequence of d dimensional vectors. If the dynamics of the system can be reduced to a set of deterministic laws, the trajectories of the system converge towards the subset of the phase space, called the attractor.

The time delay t can be defined by means of an autocorrelation function or, as used in this study, the average mutual information method (Fraser and Swinney, 1986; Adewole *et al.*, 2020; Adelaja *et al.*, 2021; Falayi *et al.*, 2022). Average mutual

information method defines how the measurements $X(t)$ at time t are connected in an information theoretic fashion to measurements $X(t+\tau)$ at time $t+\tau$ (Abarbanel, 1996; Falayi *et al.*, 2022). The average mutual information is defined as:

$$I(\tau) = \sum_{X(i), X(i+\tau)} P(X(i), X(i+\tau)) \log_2 \left[\frac{P(X(i), X(i+\tau))}{P(X(i)) P(X(i+\tau))} \right] \quad (2)$$

where i is total number of samples. $P(X(i))$ and $P(X(i+\tau))$ are individual probabilities for the measurements of $X(i)$ and $X(i+\tau)$. $P(X(i), X(i+\tau))$ is the joint probability density for measurements $P(X(i))$ and $P(X(i+\tau))$. The appropriate time delay t is defined as the first minimum of the average mutual information $I(t)$. Then the values of $X(i)$ and $X(i+\tau)$ are independent enough of each other to be useful as coordinates in a time delay vector. The variations of AMI against delay time for the stations are shown in Figures 2a and 2b for the Southern Nigeria and Northern Nigeria respectively.

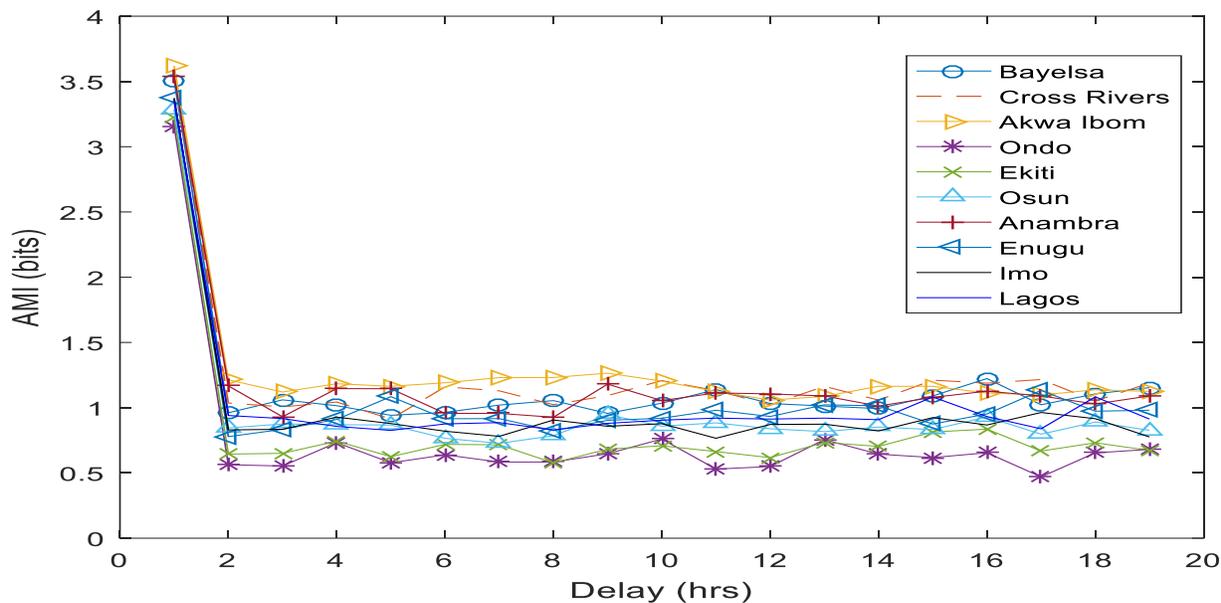


Figure 2a. Average mutual information against delay time for Bayelsa, Cross River, Akwa Ibom, Ondo, Ekiti, Osun, Anambra, Enugu, Imo and Lagos between 1901 and 2020

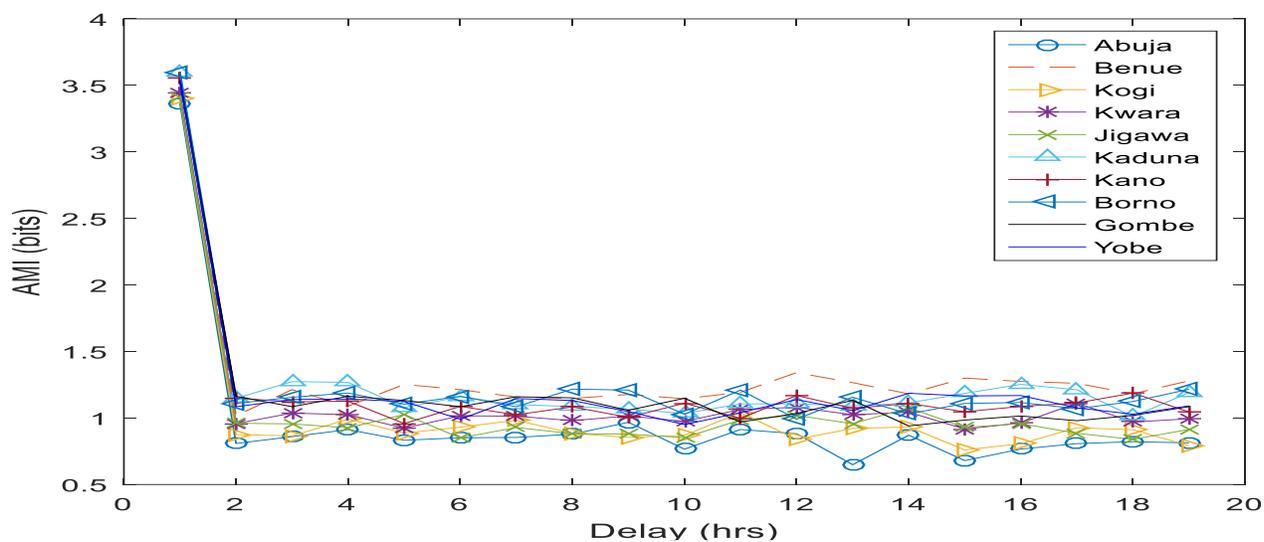


Figure 2b. Average mutual information against delay time for Abuja, Kogi, Kwara, Jigawa, Kaduna, Kano, Borno, Gombe and Yobe between 1901 and 2020.

3.3. False Nearest Neighbour (FNN)

The FNN system is used to create the precise insignificant embedding dimension to define nonlinear time series (Kennel *et al.* 1992). The section of false neighbours is produced from FNN which has a respectable linking with the detachments between samples reconstructed in embedding

dimensional spaces. When the section is insignificant for an embedding dimension, the reconstruction in embedding dimensions is enhanced. The FNN technique supports the geometric concept of phase space reconstruction using time series. Figure 3 (a and b) depicts variations of FNN against embedding dimension for the stations as

observed in the Southern Nigeria and Northern Nigeria observatory respectively. For appropriate value of the embedding

dimension the point of first minimum values are considered for all the stations.

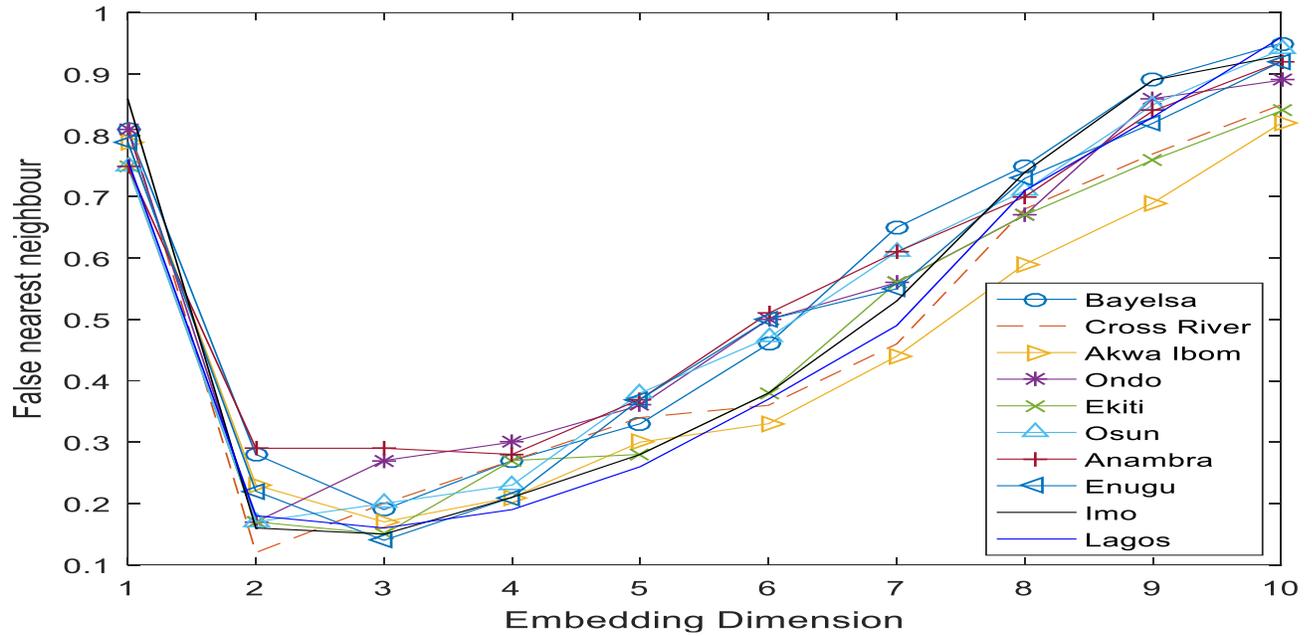


Figure 3a. Variation of False Nearest Neighbor against Embedded dimension for Bayelsa, Cross River, Akwa Ibom, Ondo, Ekiti, Osun, Anambra, Enugu, Imo and Lagos between 1901 and 2020.

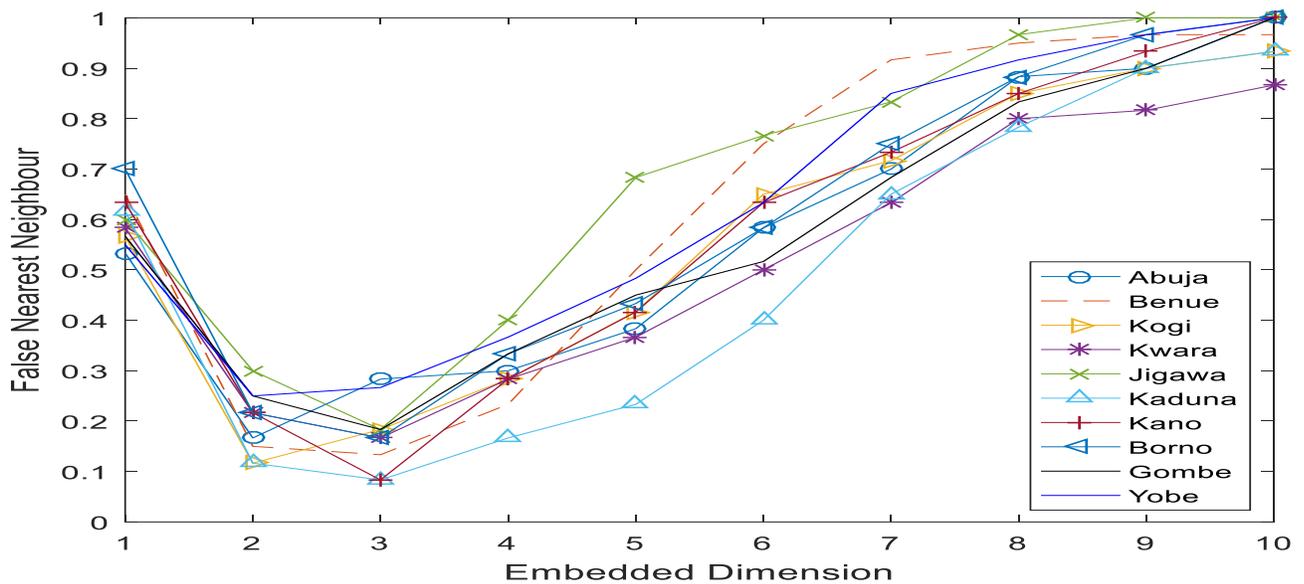


Figure 3b. Variation of False Nearest Neighbor against Embedded dimension for rainfall at Abuja, Benue, Kogi, Kwara, Jigawa, Kaduna, Kano, Borno, Gombe and Yobe between 1901 and 2020.

3.4 Lyapunov exponents

Lyapunov exponents are long-term average exponential rates of divergence or convergence of neighboring state in the phase space. A system is said to be chaotic if it has at least one positive Lyapunov exponent. The number of Lyapunov exponents in the Lyapunov exponents spectrum equals to the number degrees of freedom of the system. According to Abarbanel (1996) the local and global Lyapunov exponents can be distinguished.

Local Lyapunov exponents $\lambda_i(X^d(t), L)$,

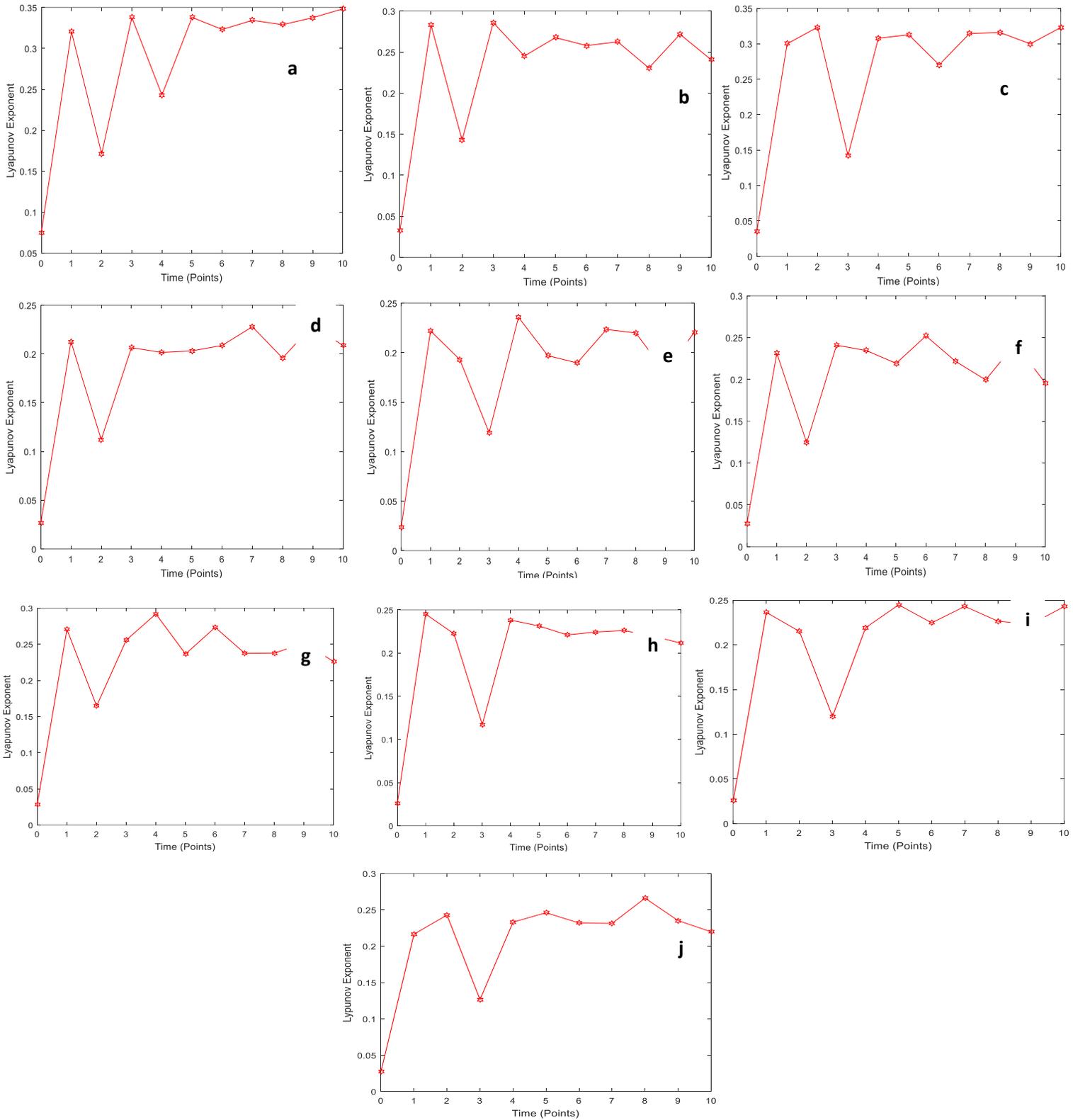
$i = 1, \dots, d$ depend on the point on the attractor $X^d(t)$ where the perturbation is initiated and the number of time steps L . When $L \rightarrow \infty$ (or in practice becomes large enough) these exponents become independent of $X^d(t)$. Average local Lyapunov exponents are obtained by averaging over the state space variable $X^d(t)$. This average local Lyapunov exponent does not depend on the initial conditions of the orbit, and converges as a power of L to the global Lyapunov exponent.

The largest global Lyapunov exponent (λ_1) determines the average horizon of global predictability for the system. Considering two points that are close in state space at

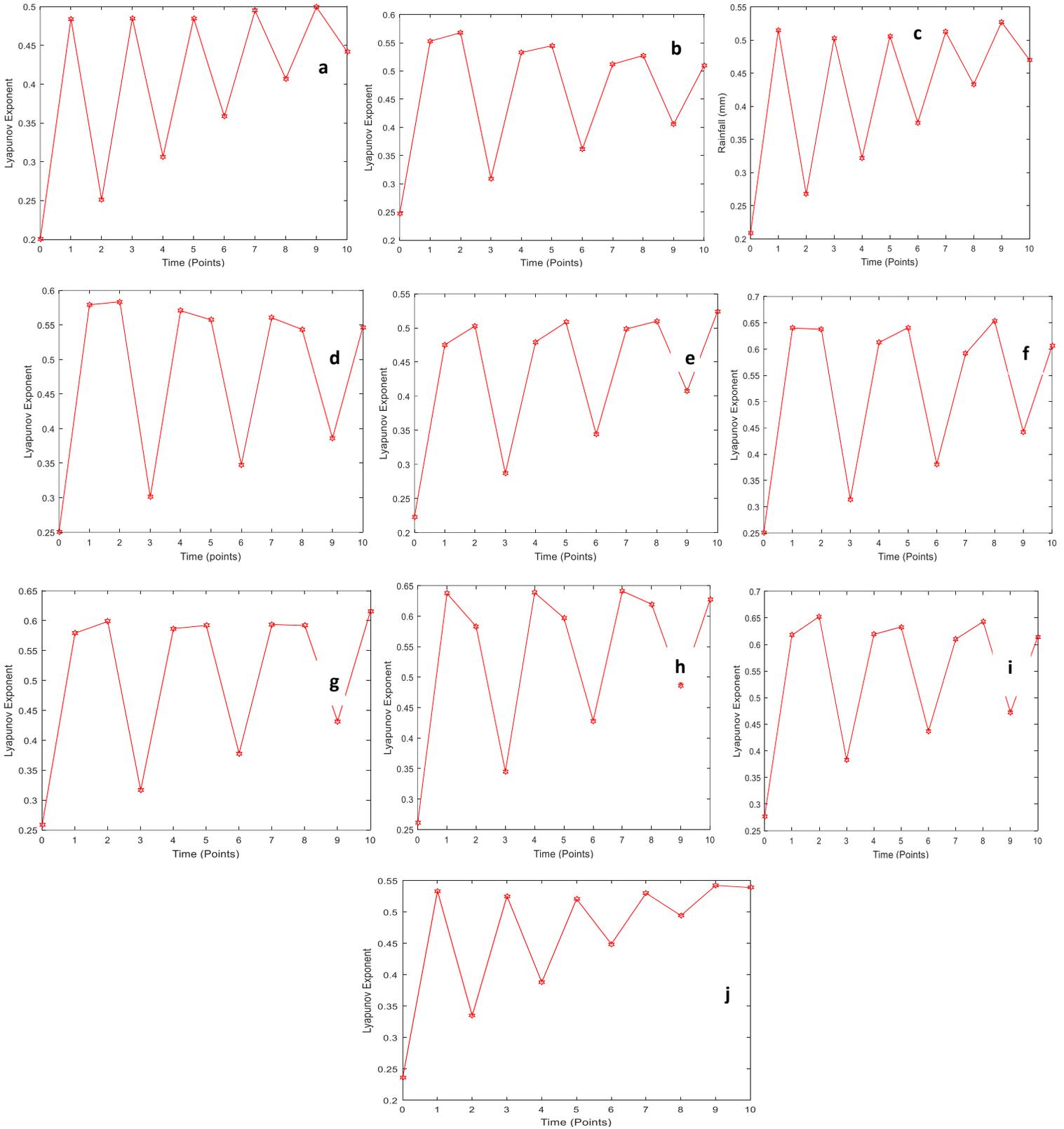
some time t , their distance at time $t' > t$ is the original distance multiplied by $e^{\lambda_1(t'-t)}$. The largest Lyapunov exponent λ_1 is defined as:

$$\lambda_1 = \frac{1}{t_M - t_0} \sum_{k=1}^M \log_2 \frac{L'(t_k)}{L(t_{k-1})} \quad (3)$$

where M is the number of replacement steps, $L(t_{k-1})$ is the Euclidean distance between the point $\{X(t_{k-1}), X(t_{k-1-t}), X(t_{k-1-2t}), \dots, X[t_{j-1-(m-1)t}]\}$ and its nearest neighbour, and $L'(t_k)$ is the evolved length of $L(t_{k-1})$ at time t_k . Before computing the largest Lyapunov exponent, the dimension d of the phase space has to be determined. In this study the largest Lyapunov exponent was computed for all the rainfall variability in all stations using dimension d obtained from the false nearest neighbors method. This enables the determination of the dimension in which the attractor is unfolded (Kennel *et al.*, 1992). Figure 4 (a & b) shows Lyapunov Exponent plot as time evolve for the Southern Nigeria and Northern Nigeria respectively. Table 2 also shows variation of the Embedding dimension and Lyapunov exponent across all the stations considered in this study while Figure 5 shows the bar chart for the variation of the Lyapunov Exponent values across all the stations.



Figures 4a: Lyapunov Exponent for rainfall at (a) Bayelsa (b) Cross River (c) Akwa-Ibom (d) Ondo (e) Ekiti (f) Osun (g) Anambra (h) Enugu (i) Imo (j) Lagos from 1901 to 2020.



Figures 4b: Lyapunov Exponent for rainfall at (a) Abuja (b) Benue (c) Kogi (d)Kwara (e) Jigawa (f) Kaduna (g) Kano (h) Borno (i) Gombe (j) Yobe States from 1901 to 2020.

Table 2: The variation of the Embedding dimension and Lyapunov exponent for various Stations.

Station Code	Embedding dimension (<i>m</i>)	Lyapunov exponent (λ)
ABJ	2	0.50
AKI	3	0.35
ANA	2	0.30
BAY	3	0.36
BNU	3	0.57
BOR	2	0.65
CRR	3	0.29
EKI	3	0.25
ENU	3	0.25
GOM	3	0.65
IMO	6	0.25
JIG	3	0.53
KAD	3	0.65
KAN	3	0.60
KOG	2	0.54
KWA	3	0.58
LAG	3	0.26
OND	2	0.25
OSU	2	0.25
YOB	2	0.55

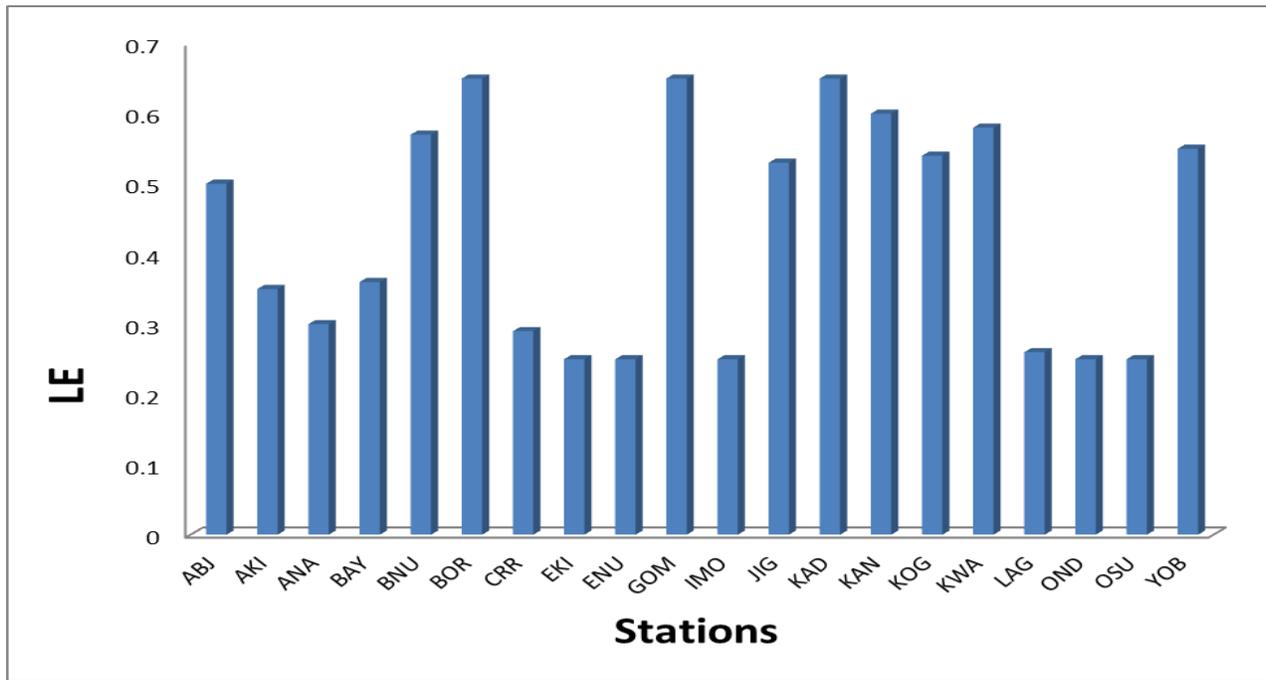


Figure 5: Variation of Lyapunov exponents of the selected stations in Nigeria.

4.0 Results and discussion

The annual rainfall investigation for 119 years displays the monthly average of rainfall for twenty stations in Nigeria (Akwa-Ibom, Balyesa, Cross River, Ondo, Ekiti, Osun, Anambra, Enugu, Imo, Lagos, Abuja, Benin, Kogi, Kwara, Jijawa, kaduna, Kano, Borno, Gombe and Yobe States respectively). The deterministic chaotic behaviour of rainfall fluctuations in Nigeria were investigated, the nonlinear method of both phase space reconstructions and Lyapunov exponent were used. The product demonstrates the potential chaotic behaviour of rainfall displaying a degree of complexity for different years during rainfall variation. Figures 1 (a - b) depicts annual rainfall at 20 stations between 1901 and 2020. Figures 1(a - b) exhibits the strength of rainfall recorded between the months of June and September (wet season). Low values of rainfall are noticed during the dry season (January, February, March, April, October, November

and December). During the wet period relative humidity is more enhanced compared to the dry period. Figure 2 (a & b) displays the actual mutual information for 20 stations as a function of a reasonable time delay. Figure 3 (a & b) demonstrates the number of false nearest neighbours (FNN) of different stations drops as the embedding dimension intensifies. The least embedding dimension relate to the lowest number of FNN. In Figures 3 (a & b), the FNN declines noticeably and the minimum rate of m linking to the smallest number of FNN can be measured as the early information for the selection of embedding dimension. When the FNN tends to zero, the chosen dimension is gotten. For the rainfall the embedding dimension is shown in Table 2 for the 20 stations. These outcomes express the residual in any dimension and it offers a numerical demonstration of how data can be exhibited in that dimension.

To ascertain the chaotic behaviour of rainfall parameter in a dynamical method, a quantifier such as Lyapunov exponent was engaged. Figures 4 (a & b) depict the Lyapunov exponent gotten from time series of the rainfall parameters measured between 1901 and 2020. These outcomes revealed that the positive values of the Lyapunov exponent for rainfall variability are displays in Table 2. The positive values indicate strong chaotic behaviour in the rainfall parameters (see Table 2). It was observed that the Lyapunov exponents is high at Gombe and Kaduna, while Osun, Ondo, Ekiti, Enugu and Imo reveal low values of the Lyapunov exponent (see Figure 5). The high latitude variations indicate high Lyapunov exponent, this implies that the rainfall variability decreases with high latitudes and the solar irradiation is also very high which is also in agreement with Falayi *et al.* (2022). The southern part of Nigeria with low latitude indicates low Lyapunov exponents, while northern part of the country with high latitudes shows high values of with high Lyapunov exponent. The inter-tropical conversion zone (ITCZ) has impact on the variability of the Lyapunov exponent. The ITCZ moves to the southern of the equator, the northern winds overcome the dry season circumstances. The ITCZ travels to the Northern, the southern westerly wind overcome to convey rain fall during the raining season. The Lyapunov exponent values varies as a result of rainfall that condenses gradually as it swings northwardly owing to the upsurge in distance of cold winds from the Atlantic Ocean. This related with the ITCZ, which is liable for the behaviour of solar heat, the greater the ITCZ impact is the smaller the occurrence of a deterministic constituent of

solar heat and in significance the lower the annual variability, leading to the variation of Lyapunov exponent. The ITCZ have impact on the stability and unpredictability conditions of rainfall owing to transformation in the degrees of freedom produced over an outside stochastic driver and can modify dynamics of the structure revealed by nonlinear dynamics.

5.0 Conclusions

Nonlinear time series of rainfall variability at twenty different stations in Nigeria were investigated in this study. It was noted that the chaotic characteristic is prominent in the rainfall data as shown from the Lyapunov exponent. It was also noted that LE ranges from 0.25 to 0.65 signifying that the rainfall variability in Nigeria is chaotic. Gombe and Kaduna which fall in the Northern part of Nigeria have the highest values of LE, while the lowest LE was observed at Ekiti, Ondo, Osun, Enugu and Imo which are all in the southern part of Nigeria. The monthly variation values of the LE can be due to the relationship between stochasticity and determinism as suggested by Unnikrishnan, (2008). The variability of the Lyapunov exponents is influenced by the inter-tropical conversion zone (ITCZ). The ITCZ moves to the southern of the equator and the northern winds overcome the dry season circumstances. The ITCZ travels to the Northern, the southern westerly wind overcome to convey rain fall during the raining season. The Lyapunov exponent values therefore varies as a result of rainfall that condenses gradually as it swings northwardly owing to the upsurge in distance of cold winds from the Atlantic Ocean.

Acknowledgments

The authors acknowledged the World Bank Data Group for providing the rainfall data of Nigeria for the duration of 120 years (1901–2015).

References

Abarbanel, H. D. I. (1996). Analysis of Observed Chaotic Data. (New York: Springer), ISBN: 978-1-4612-0763-4 (on line). pp: 13 - 93. <https://doi.org/10.1007/978-1-4612-0763-4>.

Adelaja, A. D., Laoye, J. A., Odunaike, R. K., Falayi, E. O., Adeniji, Q. A., & Ogunsanwo, F. O. (2021). Study of Total Solar Irradiance Time Series using Chaotic and Wavelet Power Spectrum Analysis. Fuw Trends in Science & Technology Journal, vol. 6, No 2, 408-414 E-Issn: 2408–5162; P-Issn: 2048–5170.

Adewole, A.T., Falayi, E.O, Roy-Layinde, T.O., Adelaja, A.D (2020). Chaotic time series analysis of meteorological parameters in some selected stations in Nigeria. Scientific African, 10, e00617. doi:10.1016/j.sciaf.2020.e00617

Agnese, C., Bagarello, V. & Nicastro, G., 2002. Alterazione di alcuni caratteri del regime pluviometrico siciliano nel periodo 1916–1999. In: Atti del Convegno Nazionale dell’AIAM, Associazione Italiana di AgroMeteorologia “L’Agrometeorologia nel Mediterraneo”. June 6-7, 2002. Catania, 18–31. Associazione Italiana di AgroMeteorologia. DOI: 10.4197/Met. 22-2.4

Brunetti, M., et al., 2004. Temperature, precipitation and extreme events during the last century in Italy. Global and Planetary Change, 40, 141-149. doi:10.1016/S0921-8181(03)00104-8

De Luis, M., et al., 2000. Spatial analysis of rainfall trends in the region of valencia (east Spain). International Journal of Climatology, 20 (12), 1451–1469. doi:10.1002/1097-0088(200010) 20:12<1451::AID-JOC547>3.0.CO;2-0.

Estrela, M., et al., 2000. Torrential events on the Spanish Mediterranean coast (Valencia region). Spatial precipitation patterns and their relation to synoptic circulation. In: Mediterranean Storms. Proceedings of the EGS Plinius Conference. Maratea, Italy, 97–106.

Falayi E.O, J. O. Adepitan, A. T. Adewole & T. O. Roy-Layinde (2022): Analysis of rainfall data of some West African countries using wavelet transform and nonlinear time series techniques, Journal of Spatial Science, DOI: 10.1080/14498596.2021.2008539

Falayi, E.O., Adewole, A.T., Adelaja, A.D., Ogundile, O.O. and Roy-Layinde, T.O. (2020). Study of nonlinear time series and wavelet power spectrum analysis using solar wind parameters and geomagnetic indices. NRIAG Journal of Astronomy and Geophysics, 9, 1, 226-237.

<https://doi.org/10.1080/20909977.2020.1728866>.

- Fraser, A.M. & Swinney, H.L. (1986). Independent coordinates for strange attractors from mutual Information, *Phys Rev A*: 33(2):1134–40.
- Kennel, M. B , Brown, R., Abarbanel, H. D. I. (1992). Determining minimum embedding dimension using a geometrical construction, *Phys. Rev. A* 45, 3403–3411.
- Lan, H., Wei, W., Soon, W., An, Z., Liu, Z., Wang, Y., Carter, R.M (2015). Dynamics of the intertropical convergence zone over the western Pacific during the Little Ice Age. *Natural geoscience*, 8(1). 315-320. DOI: [10.1038/ngeo2375](https://doi.org/10.1038/ngeo2375)
- Ojo, O. S and Ogunjo S. T (2022). Machine learning models for prediction of rainfall over Nigeria. *Scientific African* 16, e01246. <https://doi.org/10.1016/j.sciaf.2022.e01246>.
- Park, Y.R., Murray, J.T and Chen, C (1996) Predicting Sun Spots Using a Layered Perceptron Neural Network. *IEEE transactions on neural networks*, 1, 501-505.
- Poornima, U and Vinayakam, J. (2020) Hybrid SSA-ARIMA-ANN Model for Forecasting Daily Rainfall. *Water Resources Management* 34(11). DOI: [10.1007/s11269-020-02638-w](https://doi.org/10.1007/s11269-020-02638-w)
- Remya, R., & Unnikrishnan, K. (2010). Chaotic behaviour of interplanetary magnetic field under various geomagnetic conditions. *Journal of Atmospheric and Solar-Terrestrial Physics*, 72(9-10), 662-675.
- Sarp, V., Kilcik, A. A., Yurchyshyn, V., Rozelot, J. P. & Ozguc, A. (2018). Prediction of solar cycle 25: a non-linear approach. *MNRAS*. 481:2981–2985.
- Sekela, T. & Manfred, F.B., 2019. Seasonal and annual rainfall variability and their impact on rural water supply services in the Wami River Basin, Tanzania. *Water*, 11, 2055. doi:10.3390/w11102055
- Sivakumar, B. (2009). Nonlinear dynamics and chaos in hydrologic systems: latest developments and a look forward. *Stochastic Environmental Research and Risk Assessment*, 23, 1027-1036.
- Sivakumar, B., (2000). Chaos theory in hydrology: important issues and interpretations. Journal of Hydrology, 227, 1–20. doi:10.1016/S0022-1694(99)00186-9*
- Thornton, P. E., Thornton, M. M., Mayer, B. W., Wilhelmi, N., Wei, Y., Devarakonda, R., & Cook, R. B. (2014). Daymet: Daily Surface Weather Data on a 1-km Grid for North America, Version 2. Oak Ridge National Lab.(ORNL), Oak Ridge, TN (United States).
- Unnikrishnan K. (2008). Comparison of chaotic aspects of magnetosphere under various physical conditions using ae index time series, *ann. Geophys.* 26:941–953. doi:10.5194/angeo-26-941-2008.

Unnikrishnan, P., and Jothiprakash, V.(2018). Data-driven multi-time-step ahead daily rainfall forecasting using singular spectrum analysis-based data pre-processing. Journal of Hydroinformatics, 20 (3), 645-667.DOI:10.2166/hydro.2017.029

Wallace, J.M. & Hobbs, P.V., (2006). Atmospheric science: an introductory survey. Amsterdam:

Elsevier Academic Press, 316. DOI: [10.4236/ahs.2014.31005](https://doi.org/10.4236/ahs.2014.31005)

Wei, W., Jia, F., Yang, L., Chen, L., Zhang, H., Yu, Y (2014). Effects of surficial condition and rainfall intensity on runoff in a loess hilly area, China. J. Hydrol., 513 (2014), pp. 115-126, 10.1016/j.jhydrol.2014.03.022