A SIMPLIFIED RELATIONSHIP BETWEEN TWO ADOMIAN POLYNOMIAL FORMULAE

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Abstract

The Adomian decomposition method (ADM) is widely used for solving nonlinear differential and integral equations by expanding the solution into a series of Adomian polynomials. A modification of the Adomian polynomial based on the Newton-Raphson formula has been proposed to improve convergence and accuracy; however, it remains computationally demanding for higher-order terms. In this paper, we derive a simplified relationship between the standard Adomian polynomial and the Newton-Raphson-based modified Adomian polynomial for the nonlinearities of the form $(f(u) = u^m)$. It has been shown that the modified polynomial is a constant multiple of the standard polynomial, which allows direct computation of the modified polynomials from the standard ones, and confirm the simplification and demonstrate computational savings for $(m = 2)$ and $(m = 3)$.

Keywords: Adomian decomposition method, Adomian polynomials, Newton-Raphson, nonlinear operators, series solution.

1.0   Introduction

The Adomian decomposition method (ADM) is an analytic technique for obtaining series solutions of nonlinear differential and integral equations. A solution $u(x)$ is represented as and the nonlinear operator is decomposed into Adomian polynomials. Computing these polynomials for general nonlinearities can become laborious as the order increases. (Rani & Mishra, 2018) proposed a modified Adomian polynomial based on the Newton-Raphson correction, which yields improved approximations but requires additional computation. This paper derives a theorem that links the modified and standard Adomian polynomials for monomial nonlinearities $(f(u) = u^m)$, producing a direct multiplicative relationship.

Many authors have proposed modifications to ADM to accelerate convergence or simplify computation: (Adomian & Rach, 1996), (Wazwaz, 1999) (Wazwaz, 2001), (Duan, 2011), (Xie, 2013), and (Rani & Mishra, 2018) among others. Approaches include orthogonal-polynomial expansions, restarted schemes, aftertreatment techniques, and hybrid transforms. The present work complements these efforts by providing a closed-form multiplicative relation in the special but important case of power nonlinearities.

2.0 Adomian Polynomial (Adomian, 1994):

In the function;

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i(u_i)\right)\right]_{\lambda=0,n\geq 0} \qquad (1)$$

A_n is the Adomian polynomial for the nonlinear term $f(u) = u^m$

2.1 Modified Adomian Polynomial (Rani and Mishra, 2018):

In the function

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i\left(u_i - \frac{f(u_i)}{f'(u_i)}\right)\right)\right]_{\lambda=0,n\geq 0} \qquad (2)$$

$\bar{A}_n$ is the modified Adomian polynomial based on Newton-Raphson formula for the nonlinear term $f(u) = u^m$

2.2 Theorem 1

Let A_n be the Adomian polynomial for the nonlinear term $f(u) = u^m$ and let $\bar{A}_n$ be the modified Adomian polynomial based on Newton-Raphson formula for the nonlinear term $f(u) = u^m$ then

$$\bar{A}_n = \left(\frac{m-1}{m}\right)^m A_n, \quad n \geq 0, m \geq 2$$

Proof

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i\left(u_i - \frac{f(u_i)}{f'(u_i)}\right)\right)\right]_{\lambda=0,n\geq 0}$$

Let $f(u) = u^m \quad m \geq 2$

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i\left(u_i - \frac{u_i{}^m}{m(u_i)^{m-1}}\right)\right)\right]_{\lambda=0,n\geq0}$$

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i\left(\frac{m(u_i)^{m-1}u_i - u_i{}^m}{m(u_i)^{m-1}}\right)\right)\right]_{\lambda=0,n\geq0}$$

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i\left(\frac{m(u_i)^{m-1+1} - u_i{}^m}{m(u_i)^{m-1}}\right)\right)\right]_{\lambda=0,n\geq0}$$

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i\left(\frac{m(u_i)^{m} - u_i{}^m}{m(u_i)^{m-1}}\right)\right)\right]_{\lambda=0,n\geq0}$$

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i\left(\frac{(m-1)\,u_i{}^m}{m(u_i)^{m-1}}\right)\right)\right]_{\lambda=0,n\geq0}$$

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i\left(\left(\frac{m-1}{m}\right)u_i{}^{m-(m-1)}\right)\right)\right]_{\lambda=0,n\geq0}$$

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i\left(\left(\frac{m-1}{m}\right)u_i\right)\right)\right]_{\lambda=0,n\geq0}$$

If $f(u) = u^m$, then $f(b.y) = (b.y)^m = b^m.y^m = b^m f(y)$ where b is a constant.

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\left(\frac{m-1}{m}\right)^m f\left(\sum_{i=0}^{n}\lambda^i(u_i)\right)\right]_{\lambda=0,n\geq0}$$

$$\bar{A}_n = \left(\frac{m-1}{m}\right)^m \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i(u_i)\right)\right]_{\lambda=0,n\geq0}$$

$$\bar{A}_n = \left(\frac{m-1}{m}\right)^m A_n, \quad n \geq 0, m \geq 2 \qquad \text{As required} \qquad \blacksquare$$

3.0 Analysis 1

For nonlinear term $f(u) = u^2$

3.1   Adomian polynomial formula

From (1) section 2.0, we have:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i(u_i)\right)\right]_{\lambda=0,n\geq 0}$$

For $n = 0$ we have

$$A_0 = \frac{1}{0!}\frac{d^0}{d\lambda^0}\left[f\left(\sum_{i=0}^{0}\lambda^i(u_i)\right)\right]_{\lambda=0}$$

$$A_0 = (u_0)^2$$

For $n = 1$ we have

$$A_1 = \frac{1}{1!}\frac{d^1}{d\lambda^1}\left[f\left(\sum_{i=0}^{1}\lambda^i(u_i)\right)\right]_{\lambda=0}$$

$$A_1 = \frac{1}{1!}\frac{d}{d\lambda}[f(u_0 + \lambda u_1)]_{\lambda=0}$$

$$A_1 = 2u_0u_1$$

For $n = 2$ we have

$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[f\left(\sum_{i=0}^{2}\lambda^i(u_i)\right)\right]_{\lambda=0}$$

$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}[f(u_0 + \lambda u_1 + \lambda^2 u_2)]_{\lambda=0}$$

$$A_2 = \frac{2}{2!}[(u_0 + \lambda u_1 + \lambda^2 u_2)(2u_2) + (u_1 + 2\lambda u_2)(u_1 + 2\lambda u_2)]_{\lambda=0}$$

$$A_2 = 2u_0u_2 + u_1{}^2$$

For $n = 3$ we have

$$A_3 = \frac{1}{3!}\frac{d^3}{d\lambda^3}\left[f\left(\sum_{i=0}^{3}\lambda^i(u_i)\right)\right]_{\lambda=0}$$

$$A_3 = \frac{1}{3!}\frac{d^3}{d\lambda^3}[f(u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)]_{\lambda=0}$$

$$A_3 = \frac{1}{3!}\frac{d^3}{d\lambda^3}(u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)^2_{\lambda=0}$$

$$A_3 = \frac{2}{3!}[6u_0 u_3 + 2u_1 u_2 + 2u_1 u_2 + 2u_1 u_2]$$

$$A_3 = \frac{2}{3!}[6u_0 u_3 + 6u_1 u_2]$$

$$A_3 = \frac{2}{6}\,6(u_0 u_3 + u_1 u_2)$$

$$A_3 = 2(u_0 u_3 + u_1 u_2)$$

$$A_3 = 2u_0 u_3 + 2u_1 u_2$$

The first four terms of the Adomian polynomial formula for the nonlinear term $f(u) = u^2$ are thus expressed as:

$$A_0 = u_0{}^2$$

$$A_1 = 2u_0 u_1$$

$$A_2 = 2u_0 u_2 + u_1{}^2$$

$$A_3 = 2u_0 u_3 + 2u_1 u_2$$

3.2 Modified Adomian polynomial based on Newton-Raphson formula

From (2) in section 2.1, we have:

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i\left(u_i - \frac{f(u_i)}{f'(u_i)}\right)\right)\right]_{\lambda=0,n\geq0}$$

For $n = 0$ we have

$$\bar{A}_0 = \frac{1}{0!}\frac{d^0}{d\lambda^0}\left[f\left(\sum_{i=0}^{0}\lambda^i\left(u_i - \frac{f(u_i)}{f'(u_i)}\right)\right)\right]_{\lambda=0}$$

$$\bar{A}_0 = \left(\frac{u_0}{2}\right)^2$$

$$\bar{A}_0 = \left(\frac{1}{2}\right)^2 u_0{}^2$$

For $n = 1$ we have

$$\bar{A}_1 = \frac{1}{1!}\frac{d^1}{d\lambda^1}\left[f\left(\sum_{i=0}^{1}\lambda^i\left(u_i - \frac{f(u_i)}{f'(u_i)}\right)\right)\right]_{\lambda=0}$$

$$\bar{A}_1 = \frac{d}{d\lambda}f\left(\frac{u_0}{2} + \frac{\lambda u_1}{2}\right)_{\lambda=0}$$

$$\bar{A}_1 = 2\left(\frac{u_0}{2}\right)\left(\frac{u_1}{2}\right)$$

$$\bar{A}_1 = \left(\frac{1}{2}\right)^2 2u_0 u_1$$

For $n = 2$ we have

$$\bar{A}_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[f\left(\sum_{i=0}^{2}\lambda^i\left(u_i - \frac{f(u_i)}{f'(u_i)}\right)\right)\right]_{\lambda=0}$$

$$\bar{A}_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}f\left(\frac{u_0}{2} + \frac{\lambda u_1}{2} + \frac{\lambda^2 u_2}{2}\right)_{\lambda=0}$$

$$\bar{A}_2 = \frac{1}{2!}\frac{d}{d\lambda}2\left(\frac{u_0 + \lambda u_1 + \lambda^2 u_2}{2}\right)\left(\frac{u_1 + 2\lambda u_2}{2}\right)_{\lambda=0}$$

$$\bar{A}_2 = \left(\frac{1}{2}\right)^2 \frac{d}{d\lambda}(u_0 + \lambda u_1 + \lambda^2 u_2)(u_1 + 2\lambda u_2)_{\lambda=0}$$

$$\bar{A}_2 = \left(\frac{1}{2}\right)^2 [(u_0 + \lambda u_1 + \lambda^2 u_2)(2u_2) + (u_1 + 2\lambda u_2)(u_1 + 2\lambda u_2)]_{\lambda=0}$$

$$\bar{A}_2 = \left(\frac{1}{2}\right)^2 (2u_0 u_2 + u_1{}^2)$$

For $n = 3$ we have

$$\bar{A}_3 = \frac{1}{3!} \frac{d^3}{d\lambda^3} \left[f\left(\sum_{i=0}^{3} \lambda^i \left(u_i - \frac{f(u_i)}{f'(u_i)} \right) \right) \right]_{\lambda=0}$$

$$\bar{A}_3 = \frac{1}{3!} \frac{d^3}{d\lambda^3} f\left(\frac{u_0}{2} + \frac{\lambda u_1}{2} + \frac{\lambda^2 u_2}{2} + \frac{\lambda^3 u_3}{2} \right)_{\lambda=0}$$

$$\bar{A}_3 = \frac{1}{3!} \frac{d^3}{d\lambda^3} f\left(\frac{u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3}{2} \right)_{\lambda=0}$$

$$\bar{A}_3 = \frac{1}{3!} \frac{d^3}{d\lambda^3} \left(\frac{u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3}{2} \right)^2_{\lambda=0}$$

$$\bar{A}_3 = \frac{1}{3!} \frac{1}{2} [(u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)(6u_3) + (2u_2 + 6\lambda u_3)(u_1 + 2\lambda u_2 + 3\lambda^2 u_3)$$

$$+ (u_1 + 2\lambda u_2 + 3\lambda^2 u_3)(2u_2 + 6\lambda u_3)$$

$$+ (u_1 + 2\lambda u_2 + 3\lambda^2 u_3)(2u_2 + 6\lambda u_3)]_{\lambda=0}$$

$$\bar{A}_3 = \frac{1}{3!} \frac{1}{2} [6u_0 u_3 + 2u_1 u_2 + 2u_1 u_2 + 2u_1 u_2]$$

$$\bar{A}_3 = \frac{1}{6} \frac{3}{2} [2u_0 u_3 + 2u_1 u_2]$$

$$\bar{A}_3 = \left(\frac{1}{2}\right)^2 (2u_0 u_3 + 2u_1 u_2)$$

The first four terms of the modified Adomian polynomial based on Newton-Raphson formula for the nonlinear term $f(u) = u^2$ are thus expressed as:

$$\bar{A}_0 = \left(\frac{1}{2}\right)^2 u_0{}^2$$

$$\bar{A}_1 = \left(\frac{1}{2}\right)^2 2u_0 u_1$$

$$\bar{A}_2 = \left(\frac{1}{2}\right)^2 (2u_0 u_2 + u_1{}^2)$$

$$\bar{A}_3 = \left(\frac{1}{2}\right)^2 (2u_0 u_3 + 2u_1 u_2)$$

3.3 Analysis 2

For nonlinear terms $f(u) = u^3$

3.4   Adomian polynomial formula

From (1) in section 2.0, we have:

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[f\left(\sum_{i=0}^{n} \lambda^i (u_i) \right) \right]_{\lambda=0, n\geq 0}$$

For $n = 0$ we have

$$A_0 = \frac{1}{0!} \frac{d^0}{d\lambda^0} \left[f\left(\sum_{i=0}^{0} \lambda^i (u_i) \right) \right]_{\lambda=0}$$

$$A_0 = (u_0)^3$$

For $n = 1$ we have

$$A_1 = \frac{1}{1!}\frac{d^1}{d\lambda^1}\left[f\left(\sum_{i=0}^{1}\lambda^i(u_i)\right)\right]_{\lambda=0}$$

$$A_1 = 3(u_0 + \lambda u_1)^2 u_1 \quad_{\lambda=0}$$

$$A_1 = 3u_0{}^2 u_1$$

For $n = 2$ we have

$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[f\left(\sum_{i=0}^{2}\lambda^i(u_i)\right)\right]_{\lambda=0}$$

$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}[f(u_0 + \lambda u_1 + \lambda^2 u_2)] \quad_{\lambda=0}$$

$$A_2 = \frac{3}{2}(2u_0{}^2 u_2 + 2u_0 u_1{}^2)$$

$$A_2 = 3u_0{}^2 u_2 + 3u_0 u_1{}^2)$$

For $n = 3$ we have

$$A_3 = \frac{1}{3!}\frac{d^3}{d\lambda^3}\left[f\left(\sum_{i=0}^{3}\lambda^i(u_i)\right)\right]_{\lambda=0}$$

$$A_3 = \frac{1}{3!}\frac{d^3}{d\lambda^3}[f(u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)] \quad_{\lambda=0}$$

$$A_3 = \frac{3}{3!}[2(u_1 + 2\lambda u_2 + 3\lambda^2 u_3)(u_1 + 2\lambda u_2 + 3\lambda^2 u_3)^2$$
$$+ 2(u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)2(u_1 + 2\lambda u_2 + 3\lambda^2 u_3)(2u_2 + 6\lambda u_3)$$
$$+ 2(u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)(u_1 + 2\lambda u_2 + 3\lambda^2 u_3)(2u_2 + 6\lambda u_3)$$
$$+ (u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)^2(6u_3)]_{\lambda=0}$$

$$A_3 = \frac{3}{3!}[2u_1{}^3 + 6u_0u_1u_2 + 4u_0u_1u_2 + 6u_0{}^2u_3]$$

$$A_3 = \frac{3}{3!}[6u_0{}^2u_3 + 12u_0u_1u_2 + 2u_1{}^3]$$

$$A_3 = \frac{3}{6}\,2(3u_0{}^2u_3 + 6u_0u_1u_2 + u_1{}^3)$$

$$A_3 = 3u_0{}^2u_3 + 6u_0u_1u_2 + u_1{}^3$$

The first four terms of the Adomian polynomial for the nonlinear term $f(u) = u^3$ are thus expressed as:

$$A_0 = u_0{}^3$$

$$A_1 = 3u_0{}^2u_1$$

$$A_2 = 3u_0{}^2u_2 + 3u_0u_1{}^2$$

$$A_3 = 3u_0{}^2u_3 + 6u_0u_1u_2 + u_1{}^3$$

3.5   Modified Adomian polynomial based on Newton-Raphson formula

From (2) in section 2.1, we have:

$$\bar{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\left(\sum_{i=0}^{n}\lambda^i\left(u_i - \frac{f(u_i)}{f'(u_i)}\right)\right)\right]_{\lambda=0,\,n\geq 0}$$

For $n = 0$ we have

$$\bar{A}_0 = \frac{1}{0!}\frac{d^0}{d\lambda^0}\left[f\left(\sum_{i=0}^{0}\lambda^i\left(u_i - \frac{f(u_i)}{f'(u_i)}\right)\right)\right]_{\lambda=0}$$

$$\bar{A}_0 = \left(\frac{2u_0}{3}\right)^3$$

$$\bar{A}_0 = \left(\frac{2}{3}\right)^3 u_0{}^3$$

For $n = 1$ we have

$$\bar{A}_1 = \frac{1}{1!}\frac{d^1}{d\lambda^1}\left[f\left(\sum_{i=0}^{1}\lambda^i\left(u_i - \frac{f(u_i)}{f'(u_i)}\right)\right)\right]_{\lambda=0}$$

$$\bar{A}_1 = 3\left(\frac{2u_0 + 2\lambda u_1}{3}\right)^2\left(\frac{2u_1}{3}\right)_{\lambda=0}$$

$$\bar{A}_1 = 3\left(\frac{2u_0}{3}\right)^2\left(\frac{2u_1}{3}\right)$$

$$\bar{A}_1 = \left(\frac{2}{3}\right)^3 3u_0{}^2 u_1$$

For $n = 2$ we have

$$\bar{A}_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[f\left(\sum_{i=0}^{2}\lambda^i\left(u_i - \frac{f(u_i)}{f'(u_i)}\right)\right)\right]_{\lambda=0}$$

$$\bar{A}_2 = \frac{1}{2!}\left(\frac{2}{3}\right)^3\frac{d}{d\lambda}3(u_0 + \lambda u_1 + \lambda^2 u_2)^2(u_1 + 2\lambda u_2)_{\lambda=0}$$

$$\bar{A}_2 = \frac{3}{2!}\left(\frac{2}{3}\right)^3[2(u_0 + \lambda u_1 + \lambda^2 u_2)(u_1 + 2\lambda u_2)(u_1 + 2\lambda u_2)$$

$$+ (u_0 + \lambda u_1 + \lambda^2 u_2)^2(2u_2)]_{\lambda=0}$$

$$\bar{A}_2 = \frac{3}{2}\left(\frac{2}{3}\right)^3(2u_0{}^2 u_2 + 2u_0 u_1{}^2)$$

$$\bar{A}_2 = \left(\frac{2}{3}\right)^3(3u_0{}^2 u_2 + 3u_0 u_1{}^2)$$

For $n = 3$ we have

$$\bar{A}_3 = \frac{1}{3!}\frac{d^3}{d\lambda^3}\left[f\left(\sum_{i=0}^{3}\lambda^i\left(u_i - \frac{f(u_i)}{f'(u_i)}\right)\right)\right]_{\lambda=0}$$

$$\bar{A}_3 = \frac{1}{3!}\left(\frac{2}{3}\right)^3\frac{d^2}{d\lambda^2}3(u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)^2(u_1 + 2\lambda u_2 + 3\lambda^2 u_3)_{\lambda=0}$$

$$\bar{A}_3 = \frac{3}{3!}\left(\frac{2}{3}\right)^3\frac{d}{d\lambda}[2(u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)(u_1 + 2\lambda u_2 + 3\lambda^2 u_3)^2$$

$$+ (u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)^2(2u_2 + 6\lambda u_3)]_{\lambda=0}$$

$$\bar{A}_3 = \frac{3}{3!}\left(\frac{2}{3}\right)^3[2(u_1 + 2\lambda u_2 + 3\lambda^2 u_3)(u_1 + 2\lambda u_2 + 3\lambda^2 u_3)^2$$

$$+ 2(u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)2(u_1 + 2\lambda u_2 + 3\lambda^2 u_3)(2u_2 + 6\lambda u_3)$$

$$+ 2(u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)(u_1 + 2\lambda u_2 + 3\lambda^2 u_3)(2u_2 + 6\lambda u_3)$$

$$+ (u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3)^2(6u_3)]_{\lambda=0}$$

$$\bar{A}_3 = \frac{3}{3!}\left(\frac{2}{3}\right)^3[2u_1{}^3 + 6u_0 u_1 u_2 + 4u_0 u_1 u_2 + 6u_0{}^2 u_3]$$

$$\bar{A}_3 = \left(\frac{2}{3}\right)^3(3u_0{}^2 u_3 + 6u_0 u_1 u_2 + u_1{}^3)$$

The first four terms of the modified Adomian polynomial based on newton-Raphson formula for the nonlinear term $f(u) = u^3$ are thus expressed as:

$$\bar{A}_0 = \left(\frac{2}{3}\right)^3 u_0{}^3$$

$$\bar{A}_1 = \left(\frac{2}{3}\right)^3(3u_0{}^2 u_1)$$

$$\bar{A}_2 = \left(\frac{2}{3}\right)^3(3u_0{}^2 u_2 + 3u_0 u_1{}^2)$$

$$\bar{A}_3 = \left(\frac{2}{3}\right)^3 (3u_0{}^2 u_3 + 6u_0 u_1 u_2 + u_1{}^3)$$

4.0 Using Theorem 1

For Analysis 1: When $m = 2$

$$\bar{A}_n = \left(\frac{m-1}{m}\right)^m A_n \qquad \text{[Theorem 1]}$$

$$\bar{A}_0 = \left(\frac{2-1}{2}\right)^2 A_0 = \left(\frac{1}{2}\right)^2 u_0{}^2$$

$$\bar{A}_3 = \left(\frac{2-1}{2}\right)^2 A_3 = \left(\frac{1}{2}\right)^2 2u_0 u_3 + 2u_1 u_2$$

For Analysis 2: when $m = 3$

$$\bar{A}_n = \left(\frac{m-1}{m}\right)^m A_n \qquad \text{[Theorem 1]}$$

$$\bar{A}_0 = \left(\frac{3-1}{3}\right)^3 A_0 = \left(\frac{2}{3}\right)^3 u_0{}^3$$

$$\bar{A}_3 = \left(\frac{3-1}{3}\right)^3 A_3 = \left(\frac{2}{3}\right)^3 (3u_0{}^2 u_3 + 6u_0 u_1 u_2 + u_1{}^3)$$

4.0 Discussion

We have shown how the Adomian polynomial formula is used to represent the nonlinear terms when solving a differential equation or integral equation. Recently, a modification of the Adomian polynomial formula, called the modified Adomian polynomial based on Newton-Raphson formula was introduced by Dimple Rani and Vinod Mishra. Our proposed simplified relationship between the Adomian polynomial formula and the modified Adomian polynomial based on Newton-Raphson formula has shown that we circumvent the tedious computational work involved finding the Adomian polynomials using the modified Adomian polynomial based on Newton-Raphson formula.

5.0 Conclusion

We derived and proved a simple multiplicative relationship between standard and Newton-Raphson-modified Adomian polynomials for monomial nonlinearities. The result simplifies computation and facilitates the use of Newton-Raphson corrections within ADM by avoiding repeated symbolic differentiation of modified arguments. Future work includes extending this

approach to polynomial nonlinearities (by linearity) and investigating whether analogous simplifications exist for other classes of nonlinear functions or for operator-valued nonlinearities.

Conflict of Interest

The authors declare that no conflict of interest exist between them that might influence the work described in this manuscript.

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