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ORDER NINE NON- HYBRID BLOCK METHOD BACKWARD DIFFERENTIATION FORMULA FOR NUMERICAL SOLUTION OF FIRST ORDER STIFF ORDINARY DIFFERENTIAL EQUATIONS.

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Abstract

This research concerns the derivation of seven –step non-hybrid block scheme of order nine (9) with error constants $2.6\times10^{-02}, -2.2\times10^{-02}, -5.0\times10^{-02}, -7.8\times10^{-02}, -1.1\times10^{-01}, -1.5\times10^{-01}, 1.0\times10^{-05}$ for solution of stiff ordinary differential equation. The method is derived through collocation and interpolation techniques. The constructed method is applied to solve two stiff first order initial value problem of ordinary differential equation. The numerical examples considered have shown that the new developed method gave better accuracy than some existing methods.

Keywords: Block Method, Non-Hybrid, Second Derivative, Backward Differentiation Formula, Initial value problem, Stiff ODEs,

1.0 Introduction

A stiff ordinary differential equation (ODE) problem in numerical analysis is one where the solution being sought varies slowly, but nearby solutions vary rapidly, requiring small step sizes for accurate numerical methods. (Akinfenwa *et al*. 2013), (Ibijola *et al* 2011), (Kumleng *et al*. 2013), are some of the authors who engaged in the construction of hybrid block methods for the solution of stiff IVPs. In a similar approach, a block multistep method from second derivative continuous multistep scheme with Chebyshev points as collocation points for solving stiff IVPs was developed by (Ehigie *et al*, 2014). The method was investigated to be convergent and A(α)-stable, therefore, the L(α)-stability of the method was established. Hence, the method is stiffly stable. The implementation of the method on some stiff problem showed better accuracy than other methods. Moreover (Chollom, *et al*, 2014) constructed a high order block implicit multistep methods. Indeed, the method is of high order and has been established to be A-stable. Result of problems using the new method was compared with result obtained from other existing methods. According to Anake (Lambert, 1973), hybrid method is not a method in its own right since special predictors were required to estimate the solution at the off-step point and the derivative function as well. A Continuous Two Step Trigonometrically-Fitted Second Order Method (TSTSOM) is used to solve an oscillatory second order problem of ordinary differential equations. According to Awoyemi (1992), continuous linear multistep methods have greater advantages over the discrete methods in that they give better error estimates, provide a simplified form of coefficients for further analytical work at different points and guarantee easy approximation of solutions at all interior points of the integration interval. Some others like Watts and Shampine (1972) proposed block implicit one step method, while Voss and Abbas (1997) worked with block predictor - corrector schemes. Gupta (1978) implemented second derivative linear multistep methods using the polynomial formulation. This was achieved by extending the idea of polynomial representation of the multi-step methods to second derivative. The new method was implemented using a predictor –corrector mode hence not self-starting thus capable of affecting the accuracy of the methods.

2.0 Derivation of seven-Step Non-Hybrid Block Method BDF

The approximate solution by a power series polynomial of the form.

$$y(x) = \alpha_0 + \alpha_1 x + \cdots + \alpha_p x^p = \sum_{j=0}^{p} a_j x^j \qquad (1)$$

With first derivative given as

$$y'(x) = \sum_{j=1}^{p} j\alpha_j x^{j-1} \quad (2)$$

With second derivative given as

$$y''(x) = \sum_{j=2}^{p} j(j-1)\alpha_j x^{j-2} \quad (3)$$

Where $p = r + s - 1$ and $\alpha's$ are real unknown parameters to be determined. In this method we interpolate (1) at points $x_{n+j},\ j = 0,1,2,3,4,5$ and collocate (2) at the point $x_{n+j},\ j = 7$ and the continuous non-hybrid linear multistep method (CNHLMM) reduces to,

$$y(x) = \alpha_0(x)y_n + \alpha_1(x)y_{n+1} + \alpha_2(x)y_{n+2} + \alpha_3(x)y_{n+3} + \alpha_4(x)y_{n+4} + \alpha_5(x)y_{n+5} + \alpha_6(x)y_{n+6} + h[\beta_0(x)f_n + \beta_1(x)f_{n+1} + \beta_2(x)f_{n+2} + \beta_3(x)f_{n+3} + \beta_4(x)f_{n+4} + \beta_5(x)f_{n+5} + \beta_6(x)f_{n+6} + \beta_7(x)f_{n+7}] + h^2[\gamma_0(x)g_n + \gamma_1(x)g_{n+1} + \gamma_2(x)g_{n+2} + \gamma_3(x)g_{n+3} + \gamma_4(x)g_{n+4} + \gamma_5(x)g_{n+5} + \gamma_6(x)g_{n+6} + \gamma_7(x)g_{n+7}] \quad (4)$$

And the D matrix becomes

$$\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 & x_n^9 & x_n^{10} \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 & x_{n+1}^9 & x_{n+1}^{10} \\ 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+2}^6 & x_{n+2}^7 & x_{n+2}^8 & x_{n+2}^9 & x_{n+2}^{10} \\ 1 & x_{n+3} & x_{n+3}^2 & x_{n+3}^3 & x_{n+3}^4 & x_{n+3}^5 & x_{n+3}^6 & x_{n+3}^7 & x_{n+3}^8 & x_{n+3}^9 & x_{n+3}^{10} \\ 1 & x_{n+4} & x_{n+4}^2 & x_{n+4}^3 & x_{n+4}^4 & x_{n+4}^5 & x_{n+4}^6 & x_{n+4}^7 & x_{n+4}^8 & x_{n+4}^9 & x_{n+4}^{10} \\ 1 & x_{n+5} & x_{n+5}^2 & x_{n+5}^3 & x_{n+5}^4 & x_{n+5}^5 & x_{n+5}^6 & x_{n+5}^7 & x_{n+5}^8 & x_{n+5}^9 & x_{n+5}^{10} \\ 1 & x_{n+6} & x_{n+6}^2 & x_{n+6}^3 & x_{n+6}^4 & x_{n+6}^5 & x_{n+6}^6 & x_{n+6}^7 & x_{n+6}^8 & x_{n+6}^9 & x_{n+6}^{10} \\ 0 & 1 & 2x_{n+6} & 3x_{n+6}^2 & 4x_{n+6}^3 & 5x_{n+6}^4 & 6x_{n+6}^5 & 7x_{n+6}^6 & 8x_{n+6}^7 & 9x_{n+6}^8 & 10x_{n+6}^9 \\ 0 & 1 & 2x_{n+7} & 3x_{n+7}^2 & 4x_{n+7}^3 & 5x_{n+7}^3 & 6x_{n+7}^5 & 7x_{n+7}^6 & 8x_{n+7}^7 & 9x_{n+7}^8 & 10x_{n+7}^9 \\ 0 & 0 & 2 & 6x_{n+6} & 12x_{n+6}^2 & 15x_{n+6}^3 & 30x_{n+6}^3 & 42x_{n+6}^5 & 56x_{n+6}^7 & 72x_{n+6}^7 & 90x_{n+6}^6 \\ 0 & 0 & 2 & 6x_{n+7} & 12x_{n+7}^2 & 15x_{n+7}^3 & 30x_{n+7}^3 & 42x_{n+7}^5 & 56x_{n+7}^7 & 72x_{n+7}^7 & 90x_{n+7}^6 \end{bmatrix}$$

Using Maple 18 software gives the columns of D^{-1} which are the elements of matrix that generate the values of continuous coefficient:

$$\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \beta_6, \beta_7, \gamma_6 \text{ } and \text{ } \gamma_7 \qquad (5)$$

Where the continuous coefficients (5) of the method are given as:

$$\alpha_0(x) = 1 - \frac{398615147x}{130215660h} + \frac{126722407x^2}{32553915h^2} - \frac{645944849x^3}{234388188h^3} + \frac{8559938213x^4}{7031645640h^4} - \frac{19890691043x^5}{56253165120h^5} + \frac{774471277x^6}{11250633024h^6} - \frac{50138669x^7}{5625316512h^7} + \frac{20741623x^8}{28126582560h^8} - \frac{1986683x^9}{56253165120h^9} + \frac{41929x^{10}}{56253165120h^{10}}$$

$$\alpha_1(x) = \frac{632209074x}{54256525h} - \frac{13030835993x^2}{542565250h^2} + \frac{11578519499x^3}{542565250h^3} - \frac{26296473032x^4}{2441543625h^4} + \frac{100010001923x^5}{298523500h^5} - \frac{165566147293x^6}{234388188000h^6} + \frac{11210105861x^7}{117194094400h^7} - \frac{479830297x^8}{58597047000h^8} + \frac{47200703x^9}{117194094400h^9} - \frac{2035231x^{10}}{234388188000h^{10}}$$

$$\alpha_2(x) = -\frac{561620115x}{17362088h} + \frac{5753588797x^2}{69448352h^2} - \frac{11735184987x^3}{1388967044h^3} + \frac{13026416377x^4}{277793408h^4} - \frac{26544857513x^5}{1666760448h^5} + \frac{1929382447x^6}{555586816h^6} - \frac{408199301x^7}{833380224h^7} + \frac{18048935x^8}{416690112h^8} - \frac{1215267x^9}{555586816h^9} + \frac{80329x^{10}}{1666760448h^{10}}$$

$$\alpha_3(x) = \frac{509785220x}{6510783h} - \frac{1390545309x^2}{6510783h^2} + \frac{4562087819x^3}{19532349h^3} - \frac{24238883788x^4}{175791141h^4} + \frac{17342839679x^5}{351582282h^5} - \frac{31594324337x^6}{2812658256h^6} + \frac{1155254129x^7}{703164564h^7} - \frac{210667279x^8}{1406329128h^8} + \frac{5459099x^9}{703164564h^9} - \frac{491929x^{10}}{2812658256h^{10}}$$

$$\alpha_4(x) = -\frac{1478188635x}{86810404h} + \frac{1038764363x^2}{2170261h^2} - \frac{1178540255x^3}{2170261h^3} + \frac{104174107225x^4}{312517584h^4} - \frac{464442917119x^5}{3750211008h^5} + \frac{109529114137x^6}{3750211008h^6} - \frac{8267939741x^7}{1875105504h^7} + \frac{775593641x^8}{1875105504h^8} - \frac{82460023x^9}{3750211008h^9} + \frac{1900201x^{10}}{3750211008h^{10}}$$

$$\alpha_5(x) = \frac{4953076506x}{10851305h} - \frac{28337807673x^2}{21702610h^2} + \frac{6575544819x^3}{4340522h^3} - \frac{1035257671x^4}{10851305h^4} + \frac{47433327979x^5}{130215660h^5} - \frac{6133412811x^6}{69448352h^6} + \frac{1427154259x^7}{104172528h^7} - \frac{171754543x^8}{130215660h^8} + \frac{12476527x^9}{173620880h^9} - \frac{1765171x^{10}}{1041725280h^{10}}$$

$$\alpha_6(x) = -\frac{443663287151x}{1302156600h} + \frac{8489639330263x^2}{8681044000h^2} - \frac{534189954846011x^3}{468776376000h^3} + \frac{202862015620073 9x^4}{281265825600h^4} - \frac{1557564758236841x^5}{5625316512000h^5} + \frac{379752769780157x^6}{5625316512000h^6} - \frac{29618572462789x^7}{2812658256000h^7} + \frac{1433484803003x^8}{1406329128000h^8} - \frac{313984619369x^9}{5625316512000h^9} + \frac{7439022169x^{10}}{5625316512000h^{10}}$$

$$\beta_6(x) = \frac{5092519873x}{21702610} - \frac{292515452747x^2}{434052200h} + \frac{6139589049053x^3}{7812939600h^2} - \frac{7777271191499x^4}{15625879200h^3} + \frac{5974773243481x^5}{31251758400h^4} - \frac{1457174418637x^6}{31251758400h^5} + \frac{113642407949x^7}{15625879200h^6} - \frac{5496920023x^8}{7812939600h^7} + \frac{1202651129x^9}{31251758400h^8} - \frac{3160481x^{10}}{6510783h^9}$$

$$\beta_7(x) = \frac{599947720x}{2170261} - \frac{174960744x^2}{2170261h} + \frac{208181940x^3}{2170261h^2} - \frac{405475768x^4}{6510783h^3} + \frac{480929092x^5}{19532349h^4} - \frac{242348477x^6}{39064698h^5} + \frac{78357445x^7}{78129396h^6} - \frac{7878409x^8}{78129396h^7} + \frac{448867x^9}{78129396h^8} - \frac{11077x^{10}}{78129396h^9}$$

$$\gamma_6(x) = -\frac{213770487xh}{2170261} + \frac{12367816993x^2}{43405220} - \frac{87358697159x^3}{260431320h} + \frac{335878486061x^4}{1562587920h^2} - \frac{261563682989x^5}{3125175840h^3} + \frac{64774755773x^6}{3125175840h^4} - \frac{5136914041x^7}{1562587920h^5} + \frac{63241493x^8}{195323490h^6} - \frac{56397101x^9}{3125175840h^7} + \frac{1360081x^{10}}{3125175840h^8}$$

$$\gamma_7(x) = -\frac{15036840xh}{2170261} + \frac{43935858x^2}{2170261} - \frac{52412778x^3}{2170261h} + \frac{34137216x^4}{2170261h^2} - \frac{13548465x^5}{2170261h^3} + \frac{27432223x^6}{17362088h^4} - \frac{1114449x^7}{4340522h^5} + \frac{225431x^8}{8681044h^6} - \frac{6465x^9}{4340522h^7} + \frac{643x^{10}}{17362088h^8}$$

Evaluating (4) at the following points at $x_n + x_{n+1}, x_{n+2} + x_{n+3} + x_{n+4} + x_{n+5}$ *and* x_{n+7} to obtain the following seven discrete method which are used as integrator.

$$y_n - \frac{5584643997}{905160050}y_{n+1} + \frac{12328904565}{579302432}y_{n+2} - \frac{993245935}{18103201}y_{n+3} + \frac{2225923635}{18103201}y_{n+4} - \frac{12144774717}{36206402}y_{n+5} + \frac{3638416855827}{14482560800}y_{n+6} = \frac{3h}{724128040}[41787921821f_{n+6} + 4998878400f_{n+7}] + \frac{3h^2}{506889628}[21702610g_n - 12367816993g_{n+6} - 878717160g_{n+7}] \quad (6)$$

$$y_n - \frac{929832877}{335909125}y_{n+1} - \frac{232720695}{10749092}y_{n+2} + \frac{158126780}{2687273}y_{n+3} - \frac{339521885}{2687273}y_{n+4} + \frac{895865427}{2687273}y_{n+5} - \frac{333208042133}{1343636500}y_{n+6} = -\frac{3h}{67181825}[3822664959f_{n+6} + 4998878400f_{n+7}] + \frac{6h^2}{13436365}[4340522g_{n+1} - 159142299g_{n+6} - 11059450g_{n+7}] \quad (7)$$

$$y_n - \frac{3906297492}{133651475}y_{n+1} + \frac{1009840905}{42768472}y_{n+2} + \frac{597037160}{5346059}y_{n+3} - \frac{1791326865}{5346059}y_{n+4} + \frac{4692416076}{5346059}y_{n+5} - \frac{694690128689}{1069211800}y_{n+6} = -\frac{9h}{53460590}[2654778949f_{n+6} + 297011200f_{n+7}] - \frac{9h^2}{5346059}[19532349g_{n+2} - 108857666g_{n+6} - 7391664g_{n+7}] \quad (8)$$

$$y_n - \frac{8835457}{454165}y_{n+1} + \frac{102402045}{363332}y_{n+2} - \frac{114121120}{272499}y_{n+3} + \frac{17842285}{90833}y_{n+4} + \frac{135413037}{90833}y_{n+5} - \frac{6207277891}{5449980}y_{n+6} = -\frac{h}{90833}[71713673f_{n+6} + 8005260f_{n+7}] + \frac{10h^2}{90833}[2170261g_{n+3} + 2950805g_{n+6} + 198531g_{n+7}] \quad (9)$$

$$y_n - \frac{12719569}{768900}y_{n+1} + \frac{25347825}{164032}y_{n+2} - \frac{7623425}{5126}y_{n+3} + \frac{37224575}{15378}y_{n+4} + \frac{2717973}{10252}y_{n+5} - \frac{5484961257}{4100800}y_{n+6} = -\frac{h}{205040}[214056933f_{n+6} + 29156800f_{n+7}] - \frac{h^2}{20504}[21702610g_{n+4} - 10134747g_{n+6} - 722440g_{n+7}] \quad (10)$$

$$y_n - \frac{415604067}{27760925}y_{n+1} + \frac{514481085}{4441748}y_{n+2} - \frac{760946620}{1110437}y_{n+3} + \frac{5463627495}{1110437}y_{n+4} - \frac{7939875177}{1110437}y_{n+5} + \frac{312408775643}{111043700}y_{n+6} = -\frac{9h}{5552185}[448148837f_{n+6} + 186608300f_{n+7}] + \frac{18h^2}{1110437}[195323490g_{n+5} + 61853353g_{n+6} + 4495410g_{n+7}] \quad (11)$$

$$y_{n+7} = \frac{100}{2170261}y_n - \frac{1372}{2170261}y_{n+1} + \frac{9261}{2170261}y_{n+2} - \frac{42875}{2170261}y_{n+3} + \frac{171500}{2170261}y_{n+4} - \frac{926100}{2170261}y_{n+5} + \frac{2959747}{2170261}y_{n+6} + \frac{1260h}{2170261}[539f_{n+5} + 643f_{n+6}] + \frac{88200h^2}{2170261}[7g_{n+6} - g_{n+7}] \quad (12)$$

Equation (6)-(12) constitute the members of a zero-stable block integrator of order $[9,9,9,9,9,9,9]^T$ with error constant

$$C_{11} = \left[\frac{2943919289}{111515718160}, -\frac{92694937}{4138400420}, -\frac{18735331}{374224130}, -\frac{4671407}{59949780}, -\frac{10247311}{94728480}, -\frac{255004753}{1710072980}, \frac{245}{23872871}\right]^T$$

3.0 Convergence Analysis of seven-Step Non-Hybrid Block Method BDF

The zero stability of the new methods are determined using the approach of Ehigie *et* *al.* (2014) in which he expressed the block method as:

Where

$$A^{(0)} = \begin{pmatrix} -\frac{5584643997}{90516050} & \frac{12328904565}{579302432} & -\frac{993245935}{18103201} & \frac{2225923635}{18103201} & -\frac{12144774717}{362064020} & \frac{3638416855827}{14482560800} & 0 \\ \frac{929832877}{335909125} & -\frac{232720695}{10749092} & \frac{158126780}{2687273} & -\frac{339521885}{2687273} & \frac{895865427}{2687273} & -\frac{333208042133}{13436365000} & 0 \\ -\frac{3906297492}{133651475} & \frac{1009840905}{42768472} & \frac{597037160}{5346059} & -\frac{1791326865}{5346059} & \frac{4692416076}{5346059} & -\frac{694690128689}{10692118000} & 0 \\ -\frac{8835457}{454165} & \frac{102402045}{363332} & -\frac{114121120}{272499} & -\frac{17842285}{90833} & \frac{135413037}{90833} & -\frac{6207277891}{5449980} & 0 \\ -\frac{12719569}{768900} & \frac{25347825}{164032} & -\frac{7623425}{5126} & \frac{37224575}{15378} & \frac{2717973}{10252} & -\frac{5484961257}{4100800} & 0 \\ -\frac{415604067}{27760925} & \frac{514481085}{4441748} & -\frac{760946620}{1110437} & \frac{5463627495}{1110437} & -\frac{7939875177}{1110437} & \frac{5484961257}{111043700} & 0 \\ \frac{1372}{2170261} & -\frac{9261}{2170261} & \frac{42875}{2170261} & -\frac{171500}{2170261} & \frac{926100}{2170261} & -\frac{2959747}{2170261} & 1 \end{pmatrix},$$

$$C^{(0)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad A^{(1)} =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{100}{2170261} \end{pmatrix}, \quad B^{(0)} = \begin{pmatrix} 0 & 0 & 0 & 0 & \\ \frac{26043132}{13436365} & 0 & 0 & 0 & \\ 0 & -\frac{175791141}{5346059} & 0 & 0 & \\ 0 & 0 & \frac{21702610}{90833} & 0 & \\ 0 & 0 & 0 & -\frac{10851305}{10252} & \\ 0 & 0 & 0 & 0 & \frac{351}{1\cdots} \\ 0 & 0 & 0 & 0 & \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{125363765463}{724128040} & \frac{374915880}{18103201} \\ 0 & 0 & 0 & 0 & 0 & -\frac{11467994877}{67181825} & -\frac{53078460}{2687273} \\ 0 & 0 & 0 & 0 & 0 & -\frac{23893010541}{53460590} & -\frac{267310080}{5346059} \\ 0 & 0 & 0 & 0 & 0 & -\frac{71713673}{90833} & -\frac{8005260}{90833} \\ 0 & 0 & 0 & 0 & 0 & -\frac{214056933}{205040} & -\frac{364460}{2563} \\ 0 & 0 & 0 & 0 & 0 & -\frac{4033339533}{5552185} & -\frac{335894940}{1110437} \\ 0 & 0 & 0 & 0 & 0 & \frac{679140}{2170261} & \frac{810180}{2170261} \end{pmatrix}, \quad B^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{32553915}{253444814} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Substituting $A^{(0)}$ and $A^{(1)}$ into (11) gives the characteristic polynomial of the block method $\rho(\lambda) = \det(\lambda A^{(0)} - A^{(1)})$

$$= det\left[\lambda\begin{pmatrix} -\frac{5584643997}{905160050} & \frac{12328904565}{579302432} & -\frac{993245935}{18103201} & \frac{2225923635}{18103201} & -\frac{12144774717}{36206402} & \frac{3638416855827}{14482560800} & 0 \\ \frac{929832877}{335909125} & -\frac{232720695}{10749092} & \frac{158126780}{2687273} & -\frac{339521885}{2687273} & \frac{895865427}{2687273} & -\frac{333208042133}{1343636500} & 0 \\ -\frac{3906297492}{133651475} & \frac{1009840905}{42768472} & \frac{597037160}{5346059} & -\frac{1791326865}{5346059} & \frac{4692416076}{5346059} & -\frac{694690128689}{1069211800} & 0 \\ -\frac{8835457}{454165} & \frac{102402045}{363332} & -\frac{114121120}{272499} & -\frac{17842285}{90833} & \frac{135413037}{90833} & -\frac{6207277891}{5449980} & 0 \\ -\frac{12719569}{768900} & \frac{25347825}{164032} & -\frac{7623425}{5126} & \frac{37224575}{15378} & \frac{2717973}{10252} & -\frac{5484961257}{4100800} & 0 \\ -\frac{415604067}{27760925} & \frac{514481085}{4441748} & -\frac{760946620}{1110437} & \frac{5463627495}{1110437} & -\frac{7939875177}{1110437} & \frac{5484961257}{111043700} & 0 \\ \frac{1372}{2170261} & -\frac{9261}{2170261} & \frac{42875}{2170261} & -\frac{171500}{2170261} & \frac{926100}{2170261} & -\frac{2959747}{2170261} & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{100}{2170261} \end{pmatrix}\right]$$

$$= det\begin{bmatrix} -\frac{5584643997}{905160050}\lambda & \frac{12328904565}{579302432}\lambda & -\frac{993245935}{18103201}\lambda & \frac{2225923635}{18103201}\lambda & -\frac{12144774717}{36206402}\lambda & \frac{3638416855827}{14482560800}\lambda & \\ \frac{929832877}{335909125}\lambda & -\frac{232720695}{10749092}\lambda & \frac{158126780}{2687273}\lambda & -\frac{339521885}{2687273}\lambda & \frac{895865427}{2687273}\lambda & -\frac{333208042133}{1343636500}\lambda & \\ -\frac{3906297492}{133651475}\lambda & \frac{1009840905}{42768472}\lambda & \frac{597037160}{5346059}\lambda & -\frac{1791326865}{5346059}\lambda & \frac{4692416076}{5346059}\lambda & -\frac{694690128689}{1069211800}\lambda & \\ -\frac{8835457}{454165}\lambda & \frac{102402045}{363332}\lambda & -\frac{114121120}{272499}\lambda & -\frac{17842285}{90833}\lambda & \frac{135413037}{90833}\lambda & -\frac{6207277891}{5449980}\lambda & \\ -\frac{12719569}{768900}\lambda & \frac{25347825}{164032}\lambda & -\frac{7623425}{5126}\lambda & \frac{37224575}{15378}\lambda & \frac{2717973}{10252}\lambda & -\frac{5484961257}{4100800}\lambda & \\ -\frac{415604067}{27760925}\lambda & \frac{514481085}{4441748}\lambda & -\frac{760946620}{1110437}\lambda & \frac{5463627495}{1110437}\lambda & -\frac{7939875177}{1110437}\lambda & \frac{5484961257}{111043700}\lambda & \\ \frac{1372}{2170261}\lambda & -\frac{9261}{2170261}\lambda & \frac{42875}{2170261}\lambda & -\frac{171500}{2170261}\lambda & \frac{926100}{2170261}\lambda & -\frac{2959747}{2170261}\lambda & \frac{21}{21} \end{bmatrix}$$

$$= \lambda^6(\lambda - 1) = 0$$

Therefore, $\lambda_1 = 1, \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 0$.The block method (11) by definition is A-stable and by Henrici (1962), the block method is convergent.

4.0 Numerical Examples

The method newly developed seven-step non-hybrid block method to solve some sample problems as shown below

Problem 1

$y_1' = -29998y_1 - 59994y_2$

$y_2' = 9999y_1 + 19997y_2$

Exact $y_1(x) = \left(\frac{1}{9999}\right)(29997e^{-10000x}-19998e^{-x})y_1(0) = 1$

$y_2(x) =$ -$e^{-10000x}$+$e^{-x}y_2(0) = 0$

$with\ h = 0.1$

Table 1: Absolute errors of numerical solutions of problem 1 solved with the method $k = 7$

x	(y_1)	(y_2)
0.0	0.00000E+00	0.00000E+00
0.1	6.25725E-16	3.12870E-16
0.2	2.39215E-16	1.19610E-16
0.3	1.07125E-16	5.35650E-17
0.4	4.18200E-17	2.09100E-17
0.5	7.85000E-18	3.92500E-18
0.6	2.50000E-19	1.25000E-19
0.7	2.20000E-19	1.10000E-19
0.8	1.95000E-19	9.50000E-20
0.9	1.55000E-19	7.50000E-20
0.1	3.50000E-20	2.00000E-20

Problem 2

$y_1' = -2y_1 + y_2 + 2sinx$

$y_2' = 998y_1 - 999y_2 + 999(cosx - sinx)$ $\quad$ $with\ h = 0.1$

Exact $y_{1(x)} = 2e^{-x} + sinx\, y_1(0) = 2$

$y_2(x) = 2e^{-x} + cosx\, y_2(0) = 3$

Table 2: Absolute errors of numerical solutions of problem 2 solved with the method $k = 7$

x	(y_1)	(y_2)
0.0	0.00000E+00	0.00000E+00
0.1	1.10722E-08	2.07278E-09
0.2	3.46033E-08	3.78904E-08
0.3	6.58280E-08	6.03810E-08
0.4	4.29188E-08	3.37453E-08
0.5	1.71001E-08	2.15657E-08
0.6	6.05325E-08	5.61850E-08
0.7	4.79938E-08	3.88298E-08
0.8	8.78740E-09	1.43422E-08
0.9	5.75325E-08	5.43710E-08
0.1	5.33985E-08	4.44273E-08

Table 3: Comparison of seven-Step Non-Hybrid Block Method BDF $K = 7$

for Problem 1

x	Error in $y_1(x)$	Proposed Method $y_2(x)$	Error in (Mohammed and Adeniyi,2014) $y_1(x)$	Error in (Olabode and Yusuph,2019) $y_1(x)$
0.1	6.25725E-16	3.12870E-16	0.00000E+00	-7.5647E-11
0.2	2.39215E-16	1.19610E-16	0.00000E+00	1.83983E-09
0.3	1.07125E-16	5.35650E-17	1.00000E-09	4.42400E-09
0.4	4.18200E-17	2.09100E-17	1.00000E-09	1.03587E-08
0.5	7.85000E-18	3.92500E-18	1.00000E-09	1.12999E-08
0.6	2.50000E-19	1.25000E- 19	1.00000E-09	1.46095E-08
0.7	2.20000E-19	1.10000E-19	9.99999E-10	2.05295E-08
0.8	1.95000E-19	9.50000E-20	1.00000E-09	1.95075E-08
0.9	1.55000E-19	7.50000E-20	2.00000E-09	1.08431E-08
0.1	3.50000E-20	2.00000E-20	1.00000E-09	1.54095E-08

5.0 Conclusions

Seven-step non-hybrid backward differentiation formula has been derived and the continuous schemes of the method have been obtained and applied to systems of linear stiff first order of ordinary differential equations. The newly constructed method is consistent, convergent and zero-stable. Two numerical problem to test the efficiency and accuracy of the method have been considered. Table 3 shows absolute error of the method compared with that of absolute error in (Mohammed and Adeniyi, 2014) and absolute error in (Olabode and Yusuph, 2019). It shows that the new method gives a better approximation than that of the existing method for the solution of ordinary differential equation.

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